From Pendulum to Pulse: A Thought-Experiment on Logostic q → τ(t), Entrainment, and Trance Music

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This paper proposes a hands-on thought experiment: treat a driven double pendulum as a living control signal that can entrain a musical system into trance-grade grooves while illustrating a broader informational dynamic, q → τ(t). Here, *q(t)* is the following state, and *τ(t)* is the moving "truth/driver" supplied by base motion. By coupling simple garage-level actuation (rotisserie motor, shaker puck) and returnable sensors (Hall switch, photogate) to a DAW, we watch the system migrate from chaos to lock and back—revealing how resonance, not coercion, yields coherent structure. The same frame models camera-only tracking (Python video) and hybrid rigs, clarifying why latency and jitter matter for feel, and why tight clocks paired with soft, evolving controllers sound most "alive." We then elevate the lab toy to Logostics: the thesis that many systems—physical, cognitive, social, spiritual—stabilize by resonant alignment to a moving generative target *τ(t)*. The pendulum becomes a pedagogical microcosm: parametric drive as revelation, phase capture as obedience, release as exploration, and informational-Lyapunov ideas as "stability while truth moves." The paper ends with falsifiable musical predictions, practical build recipes, a simulation-to-MIDI pipeline, and speculative extensions (architecture, liturgy, cities) where designed base motion could foster communal coherence without top-down force.

Keywords: double pendulum, entrainment, trance music, q → τ(t), Logostics, informational Lyapunov, parametric resonance, active base motion, hybrid sensing, video tracking, USB-MIDI, Amazon parts, latency, jitter, chaos, polyrhythm, capture-pump-release, effective potentials, generative score.

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One-Page “Do This First” (for the impatient builder)

• Mount a small shaker puck under the base; feed a 128 BPM sine from your DAW.
• Stick a magnet on the lower bob; place a Hall sensor at centerline; photogate mid-radius.
• Use the Leonardo USB-MIDI sketch to emit **kick** on quarters, **hat** on 1/16, **CC 74** from speed.
• Add light HF dither; ride its depth over 16–32 bars for **capture → pump → release**.
• Render a 60-second MIDI score from simulation; compare to the hardware groove; listen for the predicted **locked-yet-alive** feel.

Deliverables referenced (already prototyped here):
• Working Leonardo USB-MIDI sketch (Hall + photogate → notes/CC).
• Python simulator → MIDI generator (driven double pendulum → SMF).
• Concrete parameter ranges, safety notes, and build recipes using Amazon-available parts.

0. Orientation and Commitments

0.1 What this is (a thought-experiment you can actually build)

This work is a **hands-on, falsifiable thought experiment**. It has two interchangeable embodiments:

- **Software-only:** a single Python script that simulates a driven double pendulum, extracts musically meaningful events (zero-crossings, threshold entries, transient near-locks), quantizes them to a swung grid, and exports a **Standard MIDI File** playable on any commercial device or DAW. You can reproduce every figure and sonic claim with this script alone.
- **Bench-top hardware (optional):** an inexpensive, return-friendly kit (rotisserie motor or shaker "puck," Arduino Leonardo, Hall sensor, photogate, magnets). The hardware rig lets you listen to the same logic operating on real motion, with the DAW clock entraining the base and sensors supplying tight triggers and CCs.

In both modes, the pendulum is treated as a **control signal generator**. The musical mapping is deliberately simple—**quarters for kicks, swung 1/16 for hats, continuous CC for timbre**—so that differences in feel are attributable to physics (phase capture, amplitude breathing, jitter) rather than baroque post-processing. All parameters (BPM, swing, snap-windows, drive type, damping) are explicit and logged. Results are **portable** (MIDI), **transparent** (code is short and commented), and **modifiable** (swap grammars, change quantizers, alter the drive).

Commitments: clarity over cleverness; minimal moving parts; reproducibility; and concrete predictions that can be confirmed or refuted by listening tests and simple measurements (timing jitter, density, preference ratings).

0.2 What this isn't (no mystical claims, no fragile special pleading)

- **Not mysticism.** We make no metaphysical claims about chaos, trance, or "universal rhythms." The pendulum is used as a *pedagogical* generator of structured variability; any "meaning" arises from clearly stated mappings and quantization choices.
- **Not a black-box AI system.** There is no learning, no hidden dataset, and no parameter salad. The only "model" is classical mechanics plus a tiny event grammar.
- **Not an unfalsifiable aesthetic theory.** We propose concrete listening predictions (e.g., hybrid > grid-only for perceived "aliveness" at 128 BPM with 0.55–0.60 swing) and give you code to generate the stimuli and run your own A/B/X tests.

- **Not engineering sleight-of-hand.** We surface the known pitfalls: latency and jitter in video pipelines, quantizer artifacts, actuator saturation, sensor bounce. Where the method is brittle, we say so and show the mitigation (interrupts, debouncing, snap windows, or simply letting the DAW do the quantize).
- **Not a demand for hardware.** The **Python-only** path is first-class. If the hardware adds anything, it is timing tightness and tactility—not magic.

0.3 Copy-safe notation: $q(t)$, $\tau(t)$, and $D(q \parallel \tau(t))$ as a divergence proxy

We use a small, stable vocabulary:

- $\tau(t)$ ("tau of t"): the **moving driver**—the base motion or any exogenous schedule the world supplies (parametric vertical forcing, elliptical shake, chirp sweep, periodic kicks). In software, $\tau(t)$ is a selected drive recipe; on hardware, it is the shaker/rotisserie program. Intuition: *the world moves first.*
- $q(t)$: the **follower state**—the musical pattern as heard in time (kick/hat placements, CC evolution), or, one level down, the pendulum's mechanical state $(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2)$ pushed through the event grammar and grid.
- **Event grammar:** mapping from physical features to musical symbols. Minimal set here:
  - Upward zero-cross of θ_2 → kick candidate
  - Entering $|\theta_2| > r$ → hat burst
  - $|\theta_2 - 2\theta_1| < \varepsilon$ (brief near-lock) → crash/accent
  - Continuous controllers from speed/energy → CC 74 (brightness)
- **Quantizer:** converts continuous event times to a musical grid (quarters and swung 1/16). Its parameters (BPM, swing, snap windows) are explicit levers on feel.
- **Divergence proxy** $D(q \parallel \tau(t))$**:** a *copy-safe* surrogate for "how far the follower is from the moving target." In information theory, $D(\cdot\|\cdot)$ might be KL-divergence; here we keep it conceptual and operational:
  - At the **mechanical** level, use an energy-like error or phase offset relative to a driven orbit.
  - At the **musical** level, measure misalignment between intended grid locations implied by $\tau(t)$ and realized note times/intensities in $q(t)$ (e.g., average absolute timing error per bar, or cross-correlation between driver phase and hat density).

- The design goal is **non-increase on average**: $\frac{d}{dt}V(t) \leq 0$for a suitable Vderived from D, i.e., *the follower keeps up with a moving truth without being forced into rigidity.*

We keep symbols terse and definitions audible: you can hear Dshrink when the groove **locks** and swell when the system **releases** or wanders.

0.4 The "world moves first" stance: base motion as $\tau(t)$

Our stance is methodological and audible:

1. **Primacy of $\tau(t)$.** Instead of asking the pattern to *impose* order on the world, we let the world propose a motion—**base drive**—and design the follower (q) to **resonate** with it. In practice: send a 128-BPM sine (or any schedule) to the shaker; that is $\tau(t)$.
2. **Resonance over coercion.** The follower is not "nailed" to the driver. We keep *kicks* grid-tight (foundation) but allow *hats* and *CC* to **breathe** with amplitude and phase. The texture aligns statistically (divergence falls on average) while retaining life.
3. **Moving equilibria, not fixed points.** Because $\tau(t)$changes (sweeps, dither, captures), any static notion of "correct" is inadequate. We pursue **stability while truth moves**: the pattern stays coherent even as the target glides through parameter space (e.g., chirp captures into and out of Arnold tongues).
4. **Timing is part of truth.** Latency and jitter alter how $q$hears τ. Hence:
    - **Video-only** pipelines require DAW quantization and are best for **slow macros** (CC, form).
    - **Sensors + interrupts** yield **sub-ms** triggers suitable for drums.
    - **Hybrid** lets τset the clock (audio to shaker) while $q$expresses nuance (sensors for hits, video for macro), minimizing $D$where it matters (foundation) and letting it breathe where it delights (texture).
5. **Kind targets.** A well-designed $\tau(t)$helps its followers succeed. Small high-frequency dither widens lock regions ("Kapitza kindness"); gentle chirps offer adiabatic on-ramps; circular drives create reliable phase relations. In composition terms: **make the target generous** so alignment sounds inevitable, not forced.

Practical upshot. Treat your driver as **score and stage direction** at once: the shaker or simulated $g(t)$sets the scene; the pendulum's grammar improvises within it; the quantizer edits for

audience-ready clarity. If $D(q \parallel \tau(t))$declines in your meters *and* your listeners describe the loop as "locked but alive," you are in the stance we advocate.

1. The Lab Toy: A Driven Double Pendulum

1.1 Anatomy: lengths, masses, joints; upper vs. lower link roles

A **double pendulum** consists of two rigid links joined in series:

- **Upper link**: length ℓ_1, mass m_1, angle $\theta_1(t)$measured from the local vertical. Its **pivot** is the **base**, which we will intentionally move; that base motion is our driver $\tau(t)$.
- **Lower link**: length ℓ_2, mass m_2, angle $\theta_2(t)$measured from the (instantaneous) vertical. It is hinged to the tip of the upper link.

Joints are low-friction revolute hinges. If friction is not negligible, we model each joint with a small **viscous torque** $-d_i\,\dot{\theta}_i$. Small-angle natural frequencies (about the hanging equilibrium, fixed base) are on the order of

$$\omega_p \sim \sqrt{g/\ell}\text{(for a single link)},$$

so for $\ell \in [0.5,1.0]$m we expect $\omega_p$in the **1–1.5 Hz** band. The coupled system has two principal **small-angle modes**: an **in-phase** swing (both links move together) and an **anti-phase** swing (lower link lags/leads), which interact strongly once amplitudes grow.

Role split (useful for music):

- The **upper link** is the **energy gateway**. Base motion injects/removes energy primarily through θ_1, which then redistributes via the coupling.
- The **lower link** is the **expressive tip**. Its motion is larger, faster, and more prone to flips and chaotic bursts—ideal for **event detection** (kicks/hats) and **timbre control** (CC from speed).

Practical build heuristics: keep $m_2/m_1 \in [0.6,1.2]$, $\ell_2/\ell_1 \in [0.7,1.1]$, use light, stiff links, and minimize stiction; **light damping** extends chaotic windows, **heavy damping** favors clean lock-in.

1.2 Modes of excitation

We "write" the driver $\tau(t)$by **moving the base**—either in software (effective gravity modulation) or in hardware (shakers, cranks). The drive family:

Direct 1:1 horizontal drive

- **Idea:** displace the base laterally $x_b(t) = A\sin(\Omega t)$(or equivalently rock the gravity vector by a small angle).
- **Effect: direct resonance** near $\Omega \approx \omega_p$. Clean energy injection; good for stable 1:1 captures.

Parametric vertical drive (~2:1)

- **Idea:** vibrate the base vertically, effectively modulating gravity: $g(t) = g\,[1 + \Gamma\sin(\Omega t)]$.
- **Control knob:** $\Gamma = A\Omega^2/g$(dimensionless **reduced acceleration**).
- **Effect: parametric resonance**; large responses when $2\Omega \approx \omega$. Enables dramatic pumping and even **stabilization** of nontrivial postures at high Ω.

Two-tone / quasiperiodic (Lissajous)

- **Idea:** combine orthogonal drives: $x = A\sin(\Omega_1 t)$, $y = B\sin(\Omega_2 t + \varphi)$.
- **Effect:**
  - Rational ratio $\Omega_2/\Omega_1 = m/n$→ **m:n phase-locked** figures.
  - Irrational ratio → **quasiperiodic** envelopes and long, nonrepeating modulations; rich for evolving hats/CC.

Chirps (adiabatic sweeps)

- **Idea:** slowly vary frequency $\Omega(t) = \Omega_0 + \alpha t$.
- **Effect: adiabatic capture** into/through resonances; controlled entry/exit from locked or chaotic regimes. Great for “rise-and-release” musical arcs.

Kicks (impulsive)

- **Idea:** short, periodic nudges (e.g., a solenoid tap) every T.
- **Effect: stroboscopic** energy injection; standard-map–like transitions (KAM tori → stochastic layers).

High-frequency dither

- **Idea:** superpose a small, rapid oscillation (vertical or lateral) on any drive.

- **Effect:** via averaging, reshapes the effective potential (Kapitza effect), **widening lock regions** and smoothing otherwise jagged transitions—our "kindness" to the follower.

1.3 Phenomenology to expect

These are the signatures you can see, measure, and—critically—**hear** once mapped to musical events.

Phase locking

When Ω(or simple ratios of it) matches internal rhythms, phases settle: $\theta_2 - \theta_1$(1:1), or $\theta_2 - 2\theta_1$(1:2) hover near constants. **Audible result: steady hats**, stable CC contours, and predictable "crash" accents when near-locks appear/disappear.

Beats

Slight detuning produces slow **amplitude envelopes**—energy breathes in and out between links. **Audible result:** density swells/shrinks of hats and filter brightness without reprogramming.

Autoparametric transfer

Energy moves **between modes** without direct forcing (e.g., upper link pumps lower link). **Audible result:** lower-link events (hats) intensify after upper-link growth, then hand back—natural call-and-response.

Separatrix hops

At larger amplitudes, the lower link suddenly flips or jumps between dynamical regions—**rare, dramatic** events. **Audible result:** tasteful **fills** or **crashes** that punctuate otherwise steady loops.

Kapitza effective potentials (from HF dither)

Small, fast vibrations modify stability landscapes, sometimes **stabilizing inverted or near-inverted** attitudes or extending stable oscillation plateaus. **Audible result:** longer lock durations, fewer unwanted dropouts, smoother macro-shapes.

Diagnostic tools:

- **Poincaré sections** (sample at drive phase) to visualize tori → tongues → chaos.
- **Event rasters** (times of zero-crossings/thresholds) to correlate physics with note density.
- **Energy traces** to locate capture/pump/release episodes.

1.4 Practical envelopes (order-of-magnitude)

Numbers to start with; tune from here.

Frequency bands

- **Direct (horizontal/tilt) drive:** $\Omega/2\pi \in [0.6,2]$ Hz(around each link's $\sqrt{g/\ell}$).
- **Parametric (vertical) drive:** target $2\Omega \approx \omega$of the relevant mode; practically **1–4 Hz** for most builds.

Amplitudes

- **Base motion: 1–20 mm** is plenty for table-top rigs. Start at **1–5 mm**; increase only if lock windows are too narrow.
- **HF dither amplitude: < 1 mm**, but at high frequency (8–20× the small-angle ω_p).

Damping (effective)

- Represented by d_1, d_2in N·m·s/rad or by a dimensionless **quality factor** Q.
    - **Higher Q(light damping):** broader chaotic windows, richer transitions, longer decays.
    - **Lower Q(heavier damping):** faster settling, stronger phase locking, fewer surprises.

Dimensionless controls (robust across builds)

- **Reduced acceleration:** $\Gamma = A\Omega^2/g$.
    - $\Gamma \lesssim 0.05$: gentle modulation.
    - $\Gamma \approx 0.1 - 0.2$: strong parametric pumping (mind stability).
- **Drive ratio(s):** $\Omega/\omega_p$and, for two-tone, Ω_2/Ω_1. Use these to **aim** at Arnold tongues (phase-lock plateaus) instead of chasing raw Hz values.

Safety/robustness envelopes

- Keep link masses modest (few hundred grams each), joints guarded, and **end-stops soft**.
- Increase in small steps: **frequency first, then amplitude**, with dither last.
- If you see persistent **double-hits** in sensors or simulation events, increase **debounce** (time) or **snap-window** (quantizer tolerance) rather than pushing amplitude.

Operational summary:
Pick a drive family (1:1, parametric, or quasiperiodic), set $\Omega$by **dimensionless ratios** (not just Hz), begin with **tiny amplitudes** and **moderate damping**, and add **HF dither** only to widen or smooth the target region. Expect a progression—**capture → pump → release**—clearly visible in phase plots and clearly audible once mapped to **quarters (kicks)**, **swung 1/16 (hats)**, and **slow CC** for timbre.

2. The Musical Mapping: Motion as Control Data

2.1 Signals: angles $(\boldsymbol{\theta_1}, \boldsymbol{\theta_2})$, velocities, energies, events

We expose a small set of **physically meaningful observables** from the double pendulum and treat them as **compositional primitives**:

- **Angles:** $\theta_1(t)$(upper link), $\theta_2(t)$(lower link).
  Practical ranges: small-signal $|\theta| \lesssim 0.3$ rad; expressive regimes $|\theta| \in [0.3, 1.2]$; flips when $|\theta|$crosses π.
- **Angular velocities:** $\dot{\theta}_1(t)$, $\dot{\theta}_2(t)$.
  We use magnitudes $|\dot{\theta}|$for **loudness/brightness** proxies and signs for **event polarity** (e.g., upward vs. downward zero-crossings).
- **Energy-like scalars:**
  - Per-link kinetic $K_i = \frac{1}{2} I_i \dot{\theta}_i^2$(with $I_i$effective inertia)
  - Height/"potential" proxies $U_i \sim (1 - \cos\ \theta_i)$
  - **"Breath" signal:** $E_b(t) = \alpha K_1 + \beta K_2 + \gamma(U_1 + U_2)$(choose $\alpha, \beta, \gamma$to taste).
    The **time derivative** $\dot{E}_b$is a robust marker for **rises/drops**.
- **Events (discrete markers):**
  - Zero-crossings (with direction)
  - Threshold entries/exits ($|\theta| \gtrless r$)
  - Near-lock detectors (e.g., $|\theta_2 - 2\theta_1| < \varepsilon$)
  - Flips ($|\theta|$ crosses π)
    These are the **atoms** of the event grammar below.

Pre-conditioning. For musical stability, apply:

- A **mild low-pass** (~10–20 Hz cutoff) before computing | $\dot{\theta}$ |for CCs.
- **Schmitt triggers** on thresholds/zero-crossings (two levels: enter/exit) to suppress chatter.
- **Refractory windows** (e.g., 15–30 ms) after each trigger to prevent double-hits.

2.2 Event grammar

We deliberately keep the grammar small and audible. Three rules produce a surprisingly rich trance scaffold:

Rule A — Upward zero-cross of θ_2⇒ Kick candidate

- **Detector:** if θ2(t−) < 0 and θ2(t+) ≥ 0 and \dotθ2(t+) > 0 ⇒ KICK_EVENT(t*)
- **Musical mapping:**
  - **Note:** MIDI 36 (C1), Channel 10
  - **Velocity:** $v_k = \text{clip}_{30}^{127}(v_0 + k_v \mid \dot{\theta}_2(t^*) \mid)$
  - **Grid policy:** quantize to **quarters** with generous snap window (≤200 ms) and **guarantee fill** (insert a ghost if a quarter has no candidate).
- **Intuition:** preserves a rock-solid 4/4 foundation; speed-weighted velocity injects micro-dynamics tied to actual motion.

Rule B — Threshold entry | θ_2 |> r⇒ Hat burst

- **Detector:** if |θ2| crosses upward over r ⇒ HAT_EVENT(t*)
  Choose $r$near the **mid-radius** of typical orbits (e.g., $r = 0.30$–0.45 rad).
- **Musical mapping:**
  - **Note:** MIDI 42 (F#2), Channel 10
  - **Velocity:** $v_h = \text{clip}_{30}^{127}(40 + 70\,[0.7 \mid \theta_2 \mid/(\pi/2) + 0.3\,\min(1, \mid \dot{\theta}_2 \mid)])$
  - **Grid policy:** quantize to **swung 1/16** with tighter snap (≤70–90 ms).
- **Intuition:** amplitude "breath" modulates hat **density** and **brightness**—you hear swell/ebb without losing the grid.

Rule C — Near-lock | $\theta_2 - 2\theta_1$ |$< \varepsilon \Rightarrow$ Accent/Crash

- **Detector:** when the **1:2 phase relation** holds momentarily (choose $\varepsilon \approx 0.08$–0.15 rad), gated by a **lock-gap** (e.g., ≥2 s) to avoid clusters.
- **Musical mapping:**
  - **Note:** MIDI 49 (Crash), Channel 10
  - **Grid policy:** quantize to **bar downbeats** (or selected strong beats).
- **Intuition:** sparse, meaningful punctuation when the mechanism skims a simple rational relation—tasty for fills/drops.

Extendability. Add clap/rim rules using **threshold exit** or **flip** events; add tom rolls when | $\dot{E}_b$ |spikes; or map **downward zero-crossing** to an offbeat/snare for breaks.

Debounce & priority. If multiple events land inside one grid slot, prioritize **Crash > Kick > Hat**, then **merge** or **delay** the rest to the next admissible slot.

2.3 Continuous controllers

Discrete hits give **structure**; continuous controllers give **life**. We recommend **one** primary CC and optionally one or two auxiliaries:

CC 74 "Brightness" from speed

- **Signal:** $s(t) = \alpha$ | $\dot{\theta}_1$ | $+\beta$ | $\dot{\theta}_2$ |(e.g., $\alpha = 0.6, \beta = 0.4$).
- **Sampling:** at every swung 1/16 grid tick; optionally pre-smooth with a 10–20 Hz one-pole filter.
- **Scaling:** $\mathrm{cc}_{74} = \mathrm{round}\,(10 + 110\,\mathrm{sat}\,(s)) \in [10{,}120]$.
- **Targets:** pad/bass **filter cutoff**, wavefold amount, wavetable position, or reverb modulation depth.

Optional CCs

- **Riser density / FX send from $\dot{E}_b$:** map $\max\,(0, \dot{E}_b)$to **noise burst density**, **delay send**, or **sidechain depth**.
- **Pan or stereo width from θ_1:** slow image drift that mirrors the upper link's gentle sway.

Clipping discipline. Always reserve **headroom** (e.g., 10–120 instead of 0–127) so manual performance and automation can still speak above the controller.

2.4 Quantization philosophy for Trance

Goal: tight foundation + living texture. We keep the kick absolutely dependable, let hats shimmer with swing and breath, and let CCs evolve organically. That combination yields loops listeners describe as **"locked but alive."**

Grids & windows

- **Tempo:** 128 BPM default (adjust freely).
- **Kick grid: quarters**; snap window **≤200 ms**; "guarantee fill" so no quarter is empty.
- **Hat grid: swung 1/16**; swing **0.54–0.60** (start **0.57**); snap **≤70–90 ms**.
- **CC grid: each 1/16**, same swing as hats.

Swing construction (copy-safe)

For each **pair** of straight 1/16 positions (t_{even}, t_{odd})inside a beat, place the **odd** at

$$t_{odd}^{(\text{swung})} = t_{even} + s\,(t_{even+2} - t_{even}), s \in [0.50,0.66].$$

- **Lower s(0.52–0.54):** tighter, techno-ish.
- **Higher s(0.58–0.60):** looser, psy/rolling.

Snap windows & human feel

- **Wide windows** erase nuance but maximize stability; **narrow windows** preserve microtiming but risk missed hits.
- Recommended **balanced defaults:** 200 ms (kick), 80 ms (hat), 250 ms (crash-to-bar).
- If the loop feels **stiff**, increase swing or widen the hat snap **slightly**; if it feels **messy**, reduce swing or tighten snaps.

Velocity & density management

- Use kinematic **magnitudes** for velocity (already non-negative) to avoid polarity artifacts.
- If hats become too busy at large amplitudes, introduce a **probability gate** that tapers density with recent hat count or enforce a **minimum inter-hat time** (e.g., ≥1/24 note).

Why it works (psychoacoustic rationale)

- **Predictive motor entrainment** locks to the **kick**—our error-correcting clock.
- **Swinged hats** inject **non-isochronous** microstructure that keeps auditory attention engaged without disrupting the pulse.
- **Evolving CCs** provide **slow timbral narratives** (rises, openings, breath) that read as "human agency," even though they're physics-derived.
- Together, these satisfy two competing listener needs: **temporal certainty** (foundation) and **novelty** (texture).

Quick, practical defaults (ready-to-use)

- $r = 0.35$ rad, $\varepsilon = 0.10$ rad, lock-gap $= 2.0$ s
- Swing $= 0.57$; kick snap $= 0.20$ s; hat snap $= 0.08$ s
- CC 74 range $10 \rightarrow 120$sampled each swung 1/16
- Priority **Crash > Kick > Hat**, with merge/delay on conflict

These few rules, plus the dimensioned envelopes from §1.4, are sufficient to go from **raw motion** to a **club-usable groove** that retains the pendulum's personality.

3. Three Instrumentation Paths

3.1 Video-only Python (fastest to mount)

What you need

- Any **60–120 fps** USB camera (avoid Wi-Fi/phone mirroring).
- Bright, uniform **backdrop**; bob marked with **neon tape** or LED.
- A Python script (OpenCV + simple MIDI/OSC out).

Signal chain

1. **Capture**: lock exposure, gain, and white balance; set 60 or 120 fps; ensure shutter ≤ frame period/2 to avoid motion smearing.
2. **Track** *(two robust options)*:

- **HSV blob**: threshold on color, contour with largest area, take centroid → (x, y).
- **ArUco/AprilTag**: higher contrast, robust orientation if you print a tag.

3. **Angle & events**: map $(x, y)$to θ_2(atan2 with calibrated origin). Compute $\dot{\theta}_2$by difference; low-pass (10–20 Hz) before magnitude.
4. **Schmitt crossings**: two thresholds for each event (enter/exit) to suppress chatter:
   - **Zero-cross (kick)**: $\theta_2$sign change with $\dot{\theta}_2 > 0$.
   - **Mid-radius (hat)**: | θ_2 |: $r_{low} \rightarrow r_{high}$(e.g., 0.32→0.38 rad).
5. **Quantize**: send raw times to a 128 BPM grid (quarters for kicks; swung 1/16 for hats).
6. **Emit**: MIDI/OSC—Notes on Ch10 and CC74 on Ch2; sample CC each swung 1/16.

Latency and jitter budget

- Camera frame: **16.7 ms @60 fps**, **8.3 ms @120 fps**.
- Pipeline (capture→decode→threshold→event): typically **2–3 frames**.
- **Expect 30–50 ms** one-way at 60 fps; 15–30 ms at 120 fps.
- **Mitigations**: quantize hits (erases timing error), keep CC slow (1/16 grid), avoid USB hubs with other heavy devices.

Tuning tips

- Use **manual exposure**; flicker-free lighting.
- Prefer **bigger markers** over "smarter" filters.
- If double hits: widen Schmitt band or add 15–30 ms refractory windows.
- If hats feel "late": slightly widen hat snap window (to 90–110 ms) or increase swing 1–2%.

When to choose it

- You want **zero hardware** and quick results.
- You need **macro modulation** (pads/FX) more than drum-tightness.
- You're prototyping the **q → τ(t)** idea before building a rig.

3.2 Active motion + sensors (tightest groove)

What you need

- **Driver**: shaker puck (tactile transducer) fed from DAW sub/aux out *or* a slow **rotisserie motor** with crank.
- **Sensors**: **Hall switch** + tiny magnet on lower bob; **photointerrupter** near mid-radius.
- **Microcontroller**: **Arduino Leonardo** (native USB-MIDI).

Signal chain

1. **Base entrainment**:
    - **Shaker**: route a clean 128-BPM sine or your kick to the puck → the base vibrates precisely on grid (τ(t) = audio).
    - **Rotisserie**: steady 1–3 rpm lateral sweep; layer under shaker for slow phase bias.
2. **Edge detection (hardware)**:
    - Hall on **centerline** → robust **zero-cross** (kick).
    - Photogate at **mid-radius** → **hat** entries.
3. **Interrupts**: wire sensors to Leonardo **INT** pins; trigger on **CHANGE**; enforce **microsecond debouncing** (e.g., 15–30 ms).
4. **USB-MIDI**: emit Note 36 (kick), 42 (hat); compute CC74 from $|\dot{\theta}|$ (either differentiated Hall intervals or an extra analog sensor if installed).
5. **DAW**: quantize lightly (or not at all for kicks); add swing to hats; map CC74 to filter/pad brightness.

Timing

- Sensor → interrupt: **µs**.
- Leonardo USB-MIDI → DAW: **1–3 ms** typical.
- Net **hit timing**: sub-5 ms (club-tight) even without quantize.

Mechanical envelopes

- Shaker depth: **1–5 mm** effective base motion (adjust by amp gain).
- Rotisserie crank: **2–10 mm** at base edge.

- HF dither (optional): <1 mm at **8–20×** small-angle frequency (widens lock).

Tuning & troubleshooting

- Double-hits → increase **refractory**/debounce or move sensors slightly off the most turbulent region.
- Too few hats → lower threshold $r$or reduce damping slightly.
- Over-busy hats → add minimum inter-hat interval (≥ 1/24 note) or probabilistic thinning tied to recent density.

When to choose it

- You need **reliable, punchy drums**.
- Live performance where the kick must never smear.
- You want **repeatable** "capture → pump → release" episodes.

3.3 Hybrid (recommended)

Architecture

- **τ(t) (the world) = audio**: DAW sends a clock-true signal to the **shaker** (foundation).
- **q(t) (the follower) = sensors + video**:
    - **Sensors** handle **drum events** (kick/hat) with **sub-ms** latency.
    - **Video** contributes **slow macros** (CC for pads/FX, scene changes, riser timing).

Why it maximizes repeatability *and* evolution

- **Repeatability**: kicks come from **hardware edges** synced to DAW audio; grid is literal.
- **Evolution**: hats breathe with amplitude; CC74 from video/sensor speeds drifts on a musical timescale (bars), capturing chaotic nuance without destabilizing pulse.
- **Robustness**: if video drops frames, drums remain tight; if a sensor bounces, quantize/swing still yields musical results, and video continues the macro story.

Recommended signal plan

- **KICK**: Hall zero-cross → Note 36; **no quantize** (or 1/32 to catch strays), velocity tied to $|\dot{\theta}_2|$.

- **HAT**: Photogate mid-radius → Note 42; **1/16 swung** quantize; density gates from recent amplitude.
- **CRASH**: near-lock detector (sensor or video) → Note 49 on **bar downbeats** with 2–4 s refractory.
- **CC 74**: video-derived speed/energy sampled each swung 1/16 (10–120), optionally smoothed.
- **Optional CCs**:
    - **Riser density** from $\dot{E}_b^+$ (positive energy change).
    - **Stereo width** from slow θ_1 drift.

Implementation choreography

1. Calibrate **shaker** level so the pendulum captures at small amplitudes (1–3 mm).
2. Place **Hall** exactly on centerline; adjust **photogate** radius for desired hat density.
3. Bring **video** online, but route its data only to **CCs** (not to drums).
4. Set **swing = 0.56–0.58**; kick snap 0–50 ms; hat snap 70–90 ms.
5. Add **HF dither** if you want longer captures and fewer awkward dropouts.

Failure-safe patterns

- If video fails → CCs freeze but drums stay tight.
- If a sensor fails → temporarily quantize video-derived hits harder; the shaker still defines τ(t).
- If the shaker amp clips → reduce drive amplitude; keep swing/quantize unchanged.

When to choose it

- You want the **club-reliable feel** and the **organic shimmer** simultaneously.
- You plan to **perform**: hybrid splits responsibilities cleanly (DAW clock, sensor drums, video macros).
- You aim to **demonstrate q → τ(t)** audibly: the world proposes (audio), the follower resonates (sensors/video), the music locks yet breathes.

4. Minimal Parts

This section lists **common, low-cost parts** you can drop in a cart, plus selection criteria so you can swap brands freely. Everything here is commodity—easy to return or replace.

4.1 Motion: rotisserie motor + crank; or mini tactile transducer ("puck")

Option A — Rotisserie motor + crank (slow, steady lateral drive)

- **Rotisserie/BBQ motor**, AC or DC:
  - Speed: **2–6 RPM** (≈0.03–0.10 Hz) for slow sweeps; fine for biasing phase and gentle lock.
  - Torque: ≥ **10 kg·cm** (handles a light pendulum base and an off-center crank).
  - Choose: **DC** version (12–24 V) if you want easy speed control with a PWM driver; **AC** is plug-and-go at fixed RPM.
- **Crank** (eccentric):
  - A small **aluminum or 3D-printed arm** with a radial slot; bolt to the motor shaft/hub.
  - **Radius:** start **5–10 mm** (base motion amplitude ≈ radius).
  - Add a **ball joint** or flexible linkage to avoid binding.
- **Coupler**:
  - Shaft coupler or square-to-round adapter (rotisserie shafts are often square).
- **Pros/cons**:
  - ✅ Simple, quiet, repeatable macro-swing; good "bias" for capture.
  - ❌ Not clock-true to the DAW; best when paired with the puck (hybrid).

Option B — Mini tactile transducer ("shaker puck") + audio amp (clock-true)

- **Transducer** (a.k.a. bass shaker, exciters):
  - Impedance **4–8 Ω**, power **10–50 W**. Small "puck" formats (35–50 mm dia) are ideal.
  - Mount **under the pendulum base plate** with VHB/epoxy.

- **Amplifier**:
    - Small **Class-D** stereo amp (e.g., 2×50 W @ 4–8 Ω). Use **one channel** for the puck.
    - Accepts line-level from your audio interface. Keep gain modest.
- **Drive signal** (from DAW):
    - **Sine** or filtered **kick bus** at **128 BPM** to make base motion **clock-true** (τ(t) = audio).
    - Start with −18 dBFS tone; increase until you see **1–5 mm** base motion.
- **Isolation & mounting**:
    - Insert a **foam layer** or rubber feet under the base to decouple from the table.
    - Add **threadlocker** to screws; puck vibrations can walk fasteners loose.
- **Pros/cons**:
    - ✅ Direct DAW entrainment, instant tempo changes, HF dither possible.
    - ❌ Needs an amp and careful level setting to avoid over-excursion.

Quick choice rule: If you must pick one, pick the **puck + amp**. It gives you DAW-locked τ(t). Add the rotisserie later for a slow lateral bias if desired.

4.2 Sensing: Arduino Leonardo (native USB-MIDI), Hall switch + magnet, photogate

Microcontroller

- **Arduino Leonardo** (ATmega32u4) — **native USB-MIDI**, low-jitter USB stack.
    - Alternatives: Pro Micro (32u4), Teensy LC/3.x/4.x (more I/O; also great MIDI).
    - Cable: good **USB-A↔Micro-B** or **USB-C↔Micro-B** with **ferrite** (noise suppression).

Sensors

A) Hall effect switch (for zero-cross/kick)

- Type: **latching** or **omnipolar** digital Hall switch (open-collector or push-pull).
- Placement:
    - Affix a **3–5 mm neodymium magnet** to the **lower bob**.

    - Mount the Hall sensor on the **centerline** so the magnet passes within **5–15 mm**.
- Wiring:
    - Vcc **5 V**, GND, Digital out → **Leonardo interrupt pin** (e.g., **D2 or D3**).
    - If open-collector, add a **10 kΩ pull-up** to 5 V (or enable internal pull-up).

B) Photogate / photointerrupter (for mid-radius/hat)

- Type: **slot-type** (transmissive) photointerrupter. Make a small **flag** (tab) on the bob or use a **reflective** sensor with a white/black marker.
- Placement:
    - Position at **mid-radius** so only larger swings trip it (sets hat density).
- Wiring:
    - Vcc 5 V, GND, Digital out → **Leonardo interrupt pin** (e.g., D2/D3), or any digital pin polled at high rate.
    - If analog reflective type, feed **A0** and threshold in code.

Debounce / refractory

- Implement **Schmitt logic** in software: separate enter/exit thresholds.
- Add a **refractory** of **15–30 ms** after each trigger to prevent double hits.

Leonardo pin plan (example)

- **Hall → D2 (INT0)**, trigger on **CHANGE**.
- **Photogate → D3 (INT1)**, trigger on **CHANGE**.
- Optional **status LED** on D13.
- 5 V and GND to sensors; keep leads **short** and **twisted** where possible.

MIDI mapping (default)

- **Hall (upward crossing)** → **Note 36 (Kick)**, Ch10; velocity from $|\dot{\theta}_2|$ proxy (time since last edge).
- **Photogate (enter)** → **Note 42 (Closed hat)**, Ch10; velocity from amplitude proxy.
- **CC 74 (Brightness)** on **Ch2** each 1/16, value 10–120 from blended speed.

4.3 Optional: cycle timer + 12 V solenoid for periodic "kicks"

Why use it: to introduce **impulsive energy** at controlled intervals (e.g., 1 Hz taps) for stroboscopic pumping or to **nudge** the system into/out of lock.

Parts

- **12 V push-pull solenoid** (stroke 5–10 mm). Choose a **soft bumper** (felt/rubber) on the plunger.
- **Cycle timer** (12 V DC) with **adjustable on/off** (0.1–10 s range). Many low-cost timers have two knobs or digital settings.
- **MOSFET driver** (if you want the Leonardo to control the solenoid instead of a standalone timer):
    - N-MOSFET (e.g., logic-level), **flyback diode** (1N5819/UF4007) across the solenoid.
    - Gate via **100 Ω** resistor; **10 kΩ** gate-to-ground pulldown.
- **Power**: 12 V DC supply (2–5 A headroom).

Mounting & use

- Position the solenoid so the **plunger barely kisses** the base plate at full stroke.
- Start with **short on-time** (10–20 ms) every **1–2 s**; increase only if needed.
- If driven from the DAW (via Leonardo + MOSFET), place hits on **bar lines** for dramatic "accent pumps."

Safety

- Always use a **flyback diode**.
- Keep fingers clear; even small solenoids pinch.
- Verify the base plate can withstand repeated taps (add a **felt pad**).

4.4 Mounting discipline: clamps, soft end-stops, cable routing

Base & isolation

- Use a **rigid base** (plywood or aluminum plate, 20–30 cm square).
- Add **rubber feet** or a **sorbothane layer** to isolate from the table and control resonances.

- If using a puck: mount it **under** the base, **near center**; avoid edges to prevent rocking.

Clamps & fixtures

- **C-clamps** or quick-release clamps hold the base to your workbench if needed.
- Use **T-slot** rails or a **perfboard** pattern on the base so you can slide sensors and lock them down repeatably.

Soft end-stops

- Put **foam bumpers** (EVA/Neoprene) at the extreme swing limits so occasional separatrix hops don't slam.
- For the lower link, a **light silicone ring** around the bob can soften incidental contact.

Cable routing

- Route **sensor cables** along the **upright** or the **base perimeter**; **strain-relief** at every termination (zip ties + adhesive anchors).
- Keep **power and audio** separated from **sensor** and **USB** runs to minimize noise; cross at **right angles** if they must meet.
- Add **ferrite beads** to USB and sensor leads if you notice spurious triggers (EMI).

Fasteners & adhesives

- Use **threadlocker (blue)** on all vibrating assemblies.
- **VHB tape** or **epoxy** for puck mounting; screws if the puck has tabs.
- **Velcro/dual-lock** for timers/driver boards so you can reconfigure quickly.

Grounding & noise

- Star ground: bring **all returns** (amp, sensors, solenoid driver) to a **single ground point** on the base.
- If the audio amp induces sensor chatter, try:
    - Adding **RC snubbers** (e.g., 100 Ω + 100 nF) across sensor outputs (to ground) near the microcontroller.
    - Moving the amp physically further from the sensors.
    - Powering sensors and MCU from **USB only** (amp on a separate wall supply).

Lighting (if using video)

- Even, **flicker-free** light; lock camera exposure; use a **bright marker** on the bob.
- Backdrop: matte, **high contrast** (e.g., white bob on black foam board).

Envelope guardrails (start here)

- Base motion amplitude: **1–5 mm** (increase only if locks are inconsistent).
- Drive frequencies: **0.6–2 Hz** (direct); **1–4 Hz** effective for parametric (2:1 relation).
- Dither: **<1 mm** at **8–20×** small-angle frequency.
- Sensor thresholds:
    - Hall: centerline crossing with **refractory 15–30 ms**.
    - Photogate: **r = 0.30–0.45 rad** equivalent radius; swing to taste.

Pack-list (one-box build)

- Rotisserie motor (DC), hub + crank arm, small linkage.
- Mini tactile transducer (4–8 Ω, 10–50 W) + Class-D amp + short speaker cable.
- Arduino Leonardo + USB cable (with ferrite).
- Hall switch + neodymium magnet; slot-type photointerrupter + bracket.
- 12 V supply (if using DC motor/solenoid) + DC split cable.
- Optional cycle timer, 12 V solenoid, MOSFET board, flyback diode.
- Assorted: clamps, rubber feet, foam bumpers, VHB/epoxy, threadlocker, zip ties, adhesive mounts, dupont wires, heat-shrink, ferrites.

First-power checklist

1. Mechanicals tight? **Threadlocker** cured? **Bumpers** placed?
2. Electronics: **flyback diode** installed (if solenoid)? **Grounds** common?
3. Shaker: start **quiet**; verify 1–5 mm motion before raising.
4. Sensors: test in firmware—confirm **single clean edge** per crossing.
5. DAW: route click/sine to the puck; confirm **grid-true** $\tau(t)$.
6. Only then: add swing, quantize, and record your first **locked-yet-alive** loop.

5. Protocols for Entrainment

This section turns the mechanics and mapping into **repeatable session playbooks**. Each protocol specifies **inputs** (drive type & levels), **grid/quantize settings**, **what to listen for**, **how to measure**, and **how to bail out** if it misbehaves. Values are starting points—nudge them, then commit when the loop feels "locked yet alive."

5.1 Capture → Pump → Release: lock the phrase, thicken it, let it breathe

Intent. Establish a rock-solid pulse, saturate musical density/timbre by "pumping" energy into the system, then **release** to regain air and motion.

Setup.

- **Drive:** parametric (vertical) via Shaker Puck; sine @ **128 BPM**; start **−18 dBFS** (≈1–2 mm base motion), Γ ≈ **0.05–0.10**.
- **Quantize:** KICK → quarters (snap ≤ **200 ms**), HAT → swung 1/16 (swing **0.56–0.58**, snap ≤ **80–90 ms**), CRASH → bar downbeats.
- **Controllers:** CC74 from speed, sampled each 1/16, 10–120; 50–150 ms smoothing.

Procedure.

1. **Capture.** Increase shaker gain very slowly until you hear consistent hat entries and see reduced timing spread on kicks (in DAW piano-roll or event log).
2. **Pump.** Hold amplitude for **4–16 bars**; optionally add **HF dither** (tiny 12–18× ω_p) at −30 dB relative to main drive to broaden the lock plateau.
3. **Release.** Back the shaker level down by **1–2 dB** or briefly detune drive phase; keep CC74 live. The groove should keep time while hats thin out and CC breathes.

Signs of success.

- **Kicks** remain phase-stable (< **±5 ms** jitter after quantize, measured over a bar).
- **Hats** grow denser/stronger during pump; thin during release without collapsing.
- **CC74** rises during pump (brighter timbre), decays with audible "exhale."

Troubleshooting.

- **Too stiff:** raise swing to **0.59–0.60** or shorten CC smoothing.

- **Too messy:** reduce Γ or tighten hat snap to **70 ms**; add 1/24-note minimum inter-hat interval.
- **No capture:** increment amplitude in **0.5 dB** steps; verify base motion 1–3 mm; check damping—not too high.

Variation. "Double pump": two short pumps (4 bars each) separated by 2 bars release—great before a drop.

5.2 Polyrhythm bloom: radius-placed gates drift 3:2, 5:4, 7:6 with amplitude

Intent. Use **amplitude-dependent** crossings to let implied subdivisions drift across rational ratios, generating **living polyrhythms** over a 4/4 bed.

Setup.

- **Drive:** direct (horizontal/tilt) or two-tone quasiperiodic; modest amplitude (2–4 mm).
- **Sensors/Rules:** place **two** hat gates at different radii: $r_1 \approx$ **0.30 rad**, $r_2 \approx$ **0.45 rad**; both mapped to hats or split to hat/clap.
- **Quantize:** hats to swung 1/16; optionally route r_2 to **offbeat** or **ghost** channel with −6 dB.

Procedure.

1. Start at low amplitude (mostly r_1 triggers).
2. Sweep amplitude up until r_2 begins to **enter intermittently**.
3. Let the system breathe for **16–64 bars**; listen for **phase drift** between r_1 and r_2 streams.

Audible outcomes.

- At certain amplitude bands the **inter-onset intervals** between r_1 and r_2 approximate **3:2**, **5:4**, **7:6**, etc., for several bars before sliding onward.
- You get a **rolling counter-grid** that never quite repeats, yet never loses the 4/4 anchor.

Measuring the bloom.

- Log event times for each gate; compute **beat-relative phase** $\phi_i = \text{frac}(t_i$ / 1/8-note).
- Look for ϕ-runs where the **phase difference** stays near rational multiples for **≥ 2 bars**.

Guidance knobs.

- **Spread** ($r_2 - r_1$): larger spread → rarer but more pronounced blooms.
- **Swing**: higher swing exaggerates human feel; lower swing tightens interactions.
- **Damping**: slightly lower damping increases excursion and makes r_2 participate more often.

Troubleshooting.

- **Cluttered hats:** probabilistically thin r_2 (e.g., 70% chance) or add min inter-event 1/24.
- **No blooms:** raise amplitude 0.5–1 mm or add gentle two-tone drive ($\Omega_2/\Omega_1 \approx 5/4$ or 7/6).

5.3 Noise-assisted order: small HF dither expands the lock region

Intent. Exploit the **Kapitza effect**: tiny high-frequency vibration reshapes effective potentials to **widen capture** and **smooth transitions**.

Setup.

- **Primary drive:** parametric or direct, set to just *below* reliable capture (Γ at lower edge).
- **HF dither:** add a sinusoid at **10–20×** the pendulum's small-angle frequency, amplitude **< 1 mm** (−15 to −25 dB vs. primary). If using audio, send a separate HF band from the DAW.

Procedure.

1. Establish baseline: measure capture probability over **10 trials** (8-bar ramps).
2. Add HF dither; repeat trials with identical ramps.
3. Compare **time-to-capture**, **capture duration**, and **dropout count**.

Expected effect.

- Lock occurs earlier and persists longer; **dropouts** decrease by **30–70%** in the sweet zone.
- Subjectively: groove feels **steadier**, with fewer awkward gaps, yet hats/CC still breathe.

Tuning rules.

- If it feels **buzzy/harsh**, the HF amplitude is too high—drop it by **3–6 dB**.

- If nothing changes, raise HF frequency or amplitude slightly; ensure it's **well above** the musical band.

DAW mapping.

- Tie dither depth to a macro knob; automate slight increases during **builds**, decreases during **breaks**.

Caution.

- Don't let HF dither dominate; it's a **kindness**, not the show. Keep it subtle enough to be felt in **stability**, not heard as **noise**.

5.4 Chirp captures: slow frequency sweeps to land precise orbits

Intent. Use **adiabatic sweeps** to capture specific phase relations (e.g., 1:1, 1:2) on cue—perfect for **builds**, **drops**, and **theme entrances**.

Setup.

- **Drive:** frequency-chirped shaker control $\Omega(t) = \Omega_0 + \alpha t$.
- Choose target: for parametric, aim **2Ω ≈ ω**; for direct, aim **Ω ≈ ω** (or a simple rational of a mode).
- **Chirp rates:** start **α = 0.02–0.08 Hz/s**; total sweep 2–6 Hz over 32 bars.

Procedure.

1. Start below the target tongue by **~10–20%**.
2. Sweep upward **slowly**; watch for **capture onset** (stationary phase difference).
3. Hold within the tongue for **8–16 bars** (pump), then either continue upward to **release** or sweep back down to **re-capture** at a different ratio.

Audible cues.

- **Onset of lock**: hats align, CC steadies; perceived tension resolves into drive.
- **Release**: density loosens; micro-variations reappear; ideal for introducing new elements.

Quantitative cues.

- Phase error $|\phi(t) - \phi^*|$ falls below **0.1 rad** and stays there for ≥ **4 bars**.
- Event density variance drops by **>30%** during lock vs. pre-lock.

Choreography ideas.

- **Rise → Lock → Drop**: chirp into lock across **16 bars**, crash on bar 17, cut low-end for a **break**, then unlock by pushing past the tongue.
- **Dual-tongue journey**: direct 1:1 capture (tight hats), release, parametric 1:2 capture (brighter CC), final release.

Troubleshooting.

- **Missed capture**: reduce α by 50% or extend the sweep; add tiny HF dither to widen the tongue.
- **Overlong lock (stale)**: inject a **micro-detune** (phase flip or −1 dB amplitude dip for 1 bar) to encourage release.

Quick reference (defaults to start from)

- **5.1 Capture–Pump–Release:** $\Gamma = 0.08$, swing = 0.57, hat snap = 80 ms, dither at −20 dB ($12\times \omega_p$).
- **5.2 Polyrhythm Bloom:** $r_1 = 0.32$ rad, $r_2 = 0.45$ rad, swing = 0.58, two-tone $\Omega_2/\Omega_1 \approx 5/4$ optional.
- **5.3 Noise-Assisted Order:** HF = $12–18\times \omega_p$, depth −18 to −24 dB, measure dropout rate.
- **5.4 Chirp Captures:** $\alpha = 0.02–0.08$ Hz/s over 32 bars, hold 8–16 bars inside the tongue, then release.

Use these as **launch pads**. The musical proof you've landed the protocol is simple: listeners describe the loop as **"locked but alive,"** and your logs show decreasing divergence during capture, rising variety during release, and repeatable outcomes when you rerun the same settings.

6. Simulation to Score: The MIDI Thought-Experiment

This section turns the dynamical system into a **portable musical artifact**. The pipeline is: **RK4 physics → event extraction → quantization → velocity/CC shaping → SMF (Type-1)**. Everything is reproducible, audible, and parameterized.

6.1 Physics core (RK4): double pendulum with optional parametric $g(t)$

State and parameters

- State vector $x(t) =$ $[\theta_1,$ $\dot{\theta}_1,$ $\theta_2,$.
- Link lengths (ℓ_1, ℓ_2), masses (m_1, m_2), viscous damping (d_1, d_2).
- Base drive $\tau(t)$enters as either:
  - **Parametric vertical** modulation of gravity $g(t) = g_0\,[\,1 + \Gamma \sin(2\pi f_d t)\,]$.
  - **Small tilt** $\phi(t)$of the gravity vector (emulates lateral/base rocking): $\phi(t)$can be sinusoidal, two-tone, chirped, or kicked. Copy-safe implementation: replace $\theta_i$with $\theta_i - \phi(t)$in the gravitational terms.

Equations (qualitative form)

Let $\Delta = \theta_1 - \theta_2$, $s = \sin\,\Delta$, $c = \cos\,\Delta$. With effective gravity $g(t)$and tilt $\phi(t)$, the right-hand side has the classical coupled structure:

$$\ddot{\theta}_1 = F_1(\theta_1 - \phi, \quad \theta_2 - \phi,$$
$$\ddot{\theta}_2 = F_2(\theta_1 - \phi, \quad \theta_2 - \phi,$$

where $F_1, F_2$are the standard double-pendulum coupling expressions (we keep them copy-safe here; the code in the Appendix gives a concrete instance).

Integrator

- **Runge–Kutta 4 (RK4)** at fixed step $\Delta t \in$ $[0.002, 0$ $.005]$ s(≈ 200–500 Hz).
- Stability: the above step is conservative for typical tabletop dimensions; damping helps.

Drive recipes $\tau(t)$(select one)

- parametric: $g(t)$modulated; $\Gamma \in [0.05,0.20]$, $f_d \in [0.8,1.6]$ Hz.
- tilt: $\phi(t) = A_\phi \sin(2\pi f t)$, $A_\phi \in [0.02,0.08]$rad.
- quasiperiodic: $\phi(t) = A_\phi[\sin(2\pi f_1 t) + \sin(2\pi f_2 t + \varphi)]$.
- chirp: $f(t) = f_0 + \alpha t$in either $g(t)$or $\phi(t)$.
- kicks: short duty-cycle pulses in $\phi(t)$(e.g., 80 ms every 1–2 s).

Numerical outputs

- Time series:
  $\theta_1(t)$, $\theta_2(t)$, $\dot{\theta}_1(t)$,
  .
- Optional energy proxies $K_i = \frac{1}{2} I_i \dot{\theta}_i^2$, $U_i \sim (1 - \cos\ \theta_i)$and a **breath** scalar $E_b(t) = \alpha K_1 + \beta K_2 + \gamma(U_1 + U_2)$.

6.2 Event extraction → grid quantization (quarters for kicks, swung 1/16 for hats)

Raw event detectors (sampled per integration step)

- **Kick candidate (upward zero-cross):** $\theta_2$crosses 0 with $\dot{\theta}_2 > 0$.
- **Hat candidate (mid-radius entry):** | θ_2 |crosses upward over r(e.g., $r = 0.35$rad) using a **Schmitt band** $r_{low} < r_{high}$(e.g., 0.32 → 0.38 rad) to avoid chatter.
- **Crash candidate (near 1:2 lock):** | $\theta_2 - 2\theta_1$ |< ε(e.g., $\varepsilon = 0.10$rad) with a **lock-gap** refractory of ≥ 2 s between crashes.

Debounce

- After any discrete trigger, impose a **refractory**: kicks 30 ms, hats 25 ms, crashes 2 s.

Musical grids

- Tempo BPM(default 128), PPQ (default 480).
- **Kick grid: quarters**.
- **Hat grid: swung 1/16** with swing $s \in [0.52, 0.60]$(default 0.57).
- **Crash grid: bar downbeats** (or selected strong beats).

Quantization (snap windows)

- Kick: ≤ **200 ms**; also **guarantee fill**—if a quarter lacks a candidate, place a ghost kick at grid time (lower velocity).
- Hat: ≤ **70–90 ms**; if multiple candidates fall into one slot, keep the earliest and attenuate later ones or defer to next slot.
- Crash: ≤ **250 ms** to nearest bar-downbeat; enforce lock-gap.

Velocity shaping (from physics)

- **Kick velocity** $v_k = \text{clip}\,^{127}_{30}(30 + 90 \cdot \min(1, |\,\dot{\theta}_2\,|))$.
- **Hat velocity** $v_h = \text{clip}\,^{127}_{30}(40 + 70\,[0.7 \cdot |\,\theta_2\,|/(\pi/2) + 0.3 \cdot \min(1, |\,\dot{\theta}_2\,|)])$.
- **Crash velocity**: fixed (e.g., 110) or tied to instantaneous alignment quality.

Priority & conflict resolution

- If a kick and hat quantize to the exact same tick, **kick wins** (foundation), hat is shifted to next admissible 1/16 (or attenuated).

6.3 CC stream from speed/energy at every 1/16 for living timbre

Primary CC (brightness)

- **Signal:** $s(t) = \alpha\,|\,\dot{\theta}_1\,| + \beta\,|\,\dot{\theta}_2\,|$with $\alpha = 0.6, \beta = 0.4$.
- **Pre-filter:** one-pole low-pass, 10–20 Hz cutoff, to tame jitter.
- **Sampling:** at each swung 1/16 grid time.
- **Scaling to MIDI:** $\text{CC}_{74} = \lfloor 10 + 110 \cdot \text{sat}(s) \rfloor \in [10{,}120]$.
- **Channel:** 2 (leaves Channel 10 for drums).

Optional CCs

- **Riser/FX send:** map $\max(0, \dot{E}_b)$to a density or send amount.
- **Stereo width/pan:** slow function of $\theta_1$to create spatial motion.

Design rationale

- Keeping CCs on the **same swung grid** as hats aligns timbral motion with rhythmic microfeel.
- Reserving 10…120 leaves headroom for manual performance/macros.

6.4 Deliverable: Standard MIDI File (Type-1), DAW/hardware-ready

Encoding choices

- **Format:** SMF **Type-1** (multitrack timing), but a single musical track is sufficient.
- **Division:** PPQ = **480** (granular enough for swing nuance).

- **Meta events:**
    - Time Signature (4/4): FF 58 04 04 02 18 08
    - Tempo: microseconds per quarter = $\lfloor 60{,}000{,}000/\text{BPM} \rfloor$
    - Optional Track Name: “Pendulum Trance”
- **Channels / mapping:**
    - **Ch 10** (drums): Kick **36**, Hat **42**, Crash **49**.
    - **Ch 2** (synth/pad): **CC 74** stream (brightness/filter).
- **Note lengths:** short on-times avoid overlap and keep drum envelopes snappy:
    - Kick: ~ **0.05 quarter**; Hat: ~ **0.03 quarter**; Crash: ~ **0.30 quarter**.

File integrity

- Events are sorted by **absolute tick**; deltas are encoded as **variable-length quantities**; the track ends with FF 2F 00.
- Deterministic build: given the same seed/params, the file is **bit-identical** across runs (no stochastic elements unless you add them).

Workflow

1. Run the simulation with chosen drive/params; log raw events.
2. Quantize and shape velocities/CCs.
3. Emit SMF Type-1.
4. Import into any DAW or hardware sequencer; map CC 74 to your synth’s filter/wavetable position for **immediate audition**.

6.5 Predicted listening result

Tagline: “Locked groove with breathing feel.” The kick is unwavering (temporal certainty), while the hats and timbre **breathe** with the system’s amplitude/phase (novelty).

What you will hear

- **Foundation:** a **4-on-the-floor** pulse that never wobbles, anchoring motor entrainment.
- **Texture: swung 1/16 hats** whose density and brightness ebb and flow as orbits grow/shrink—audible **beats** and **autoparametric transfers** without manual automation.
- **Accents:** rare, satisfying **crashes** when the system skims a rational relation (near 1:2 lock), lining up with bar downbeats after quantization.
- **Timbre arc: CC-driven filter** (or wavetable position) rises during pumping, then exhales on release—**organic risers** with no paint-by-numbers LFOs.
- **Controlled-chaos glitter:** occasional micro-irregularities (separatrix hops, transient near-locks) translate to delicate fills rather than derailments, thanks to grid discipline.

Acceptance criteria (measurable)

- Post-quantize kick timing jitter (RMS over a bar) **< 5 ms**; hats **< 15 ms** to grid.
- CC 74 variance increases during pump by **≥ 30%**, decreases on release.
- Listener descriptors in blind A/B/X: "alive," "breathing," "locked," **significantly** more often than with a static, hand-drawn automation baseline.

Failure modes & fixes

- **Stiff/robotic:** raise swing to 0.59–0.60; widen hat snap to ~90 ms; reduce CC smoothing.
- **Messy/busy:** lower Γ(or drive amplitude), tighten hat snap to ~70 ms, add a minimum inter-hat interval, or thin hats probabilistically.
- **Flat timbre:** increase CC gain (110 → 120 span) or blend in $\dot{E}_b$as a secondary CC.

Implementation crib (1-screen summary)

- **Integrator:** RK4, $\Delta t \approx$ 0.0025 s, $g(t) = g_0[1 + \Gamma \sin(2\pi f_d t)]$, optional $\phi(t)$.
- **Detectors:** $\theta_2 \uparrow$ zero-cross → Kick; $|\theta_2| > r$enter → Hat; $|\theta_2 - 2\theta_1| < \varepsilon$→ Crash (gap).
- **Quantize:** Kick → quarters (200 ms, guarantee fill); Hat → swung 1/16 (80 ms); Crash → bar (250 ms).
- **Vel/CC:** $v_k$from $|\dot{\theta}_2|$; $v_h$from $|\theta_2|$& $|\dot{\theta}_2|$; **CC74** from $0.6 |\dot{\theta}_1| + 0.4 |\dot{\theta}_2|$each 1/16.

- **SMF:** Type-1, PPQ 480, Ch10: 36/42/49, Ch2: CC74, tempo meta from BPM, end with FF 2F 00.

Run it, drop the MIDI in your rig, and you'll hear the core thesis: **coherence from resonance**—**q** aligning to a moving **τ(t)**—made legible as trance.

7. Informational Framing: q → τ(t)

7.1 Statement: coherence via resonant alignment, not imposition

The central claim is **methodological**: we obtain coherence not by forcing a follower to hit fixed targets, but by designing the follower to **resonate** with a **moving target**. We write this as

$$q \;\rightarrow\; \tau(t),$$

where $\tau(t)$is the *world's motion* (the base drive, external clock, architectural rhythm), and $q(t)$is the *follower state* (our musical pattern after event grammar and quantization). The driver **moves first**; the follower **aligns** by rules that conserve identity (tight kick grid) while remaining permeable to nuance (swinged hats, evolving CCs). In this stance:

- Order emerges from **phase relations** and **energy flow**, not from micromanaged trajectories.
- The designer's job is to choose **kind targets** $\tau(t)$(with capture plateaus, on-ramps, and exits) and **gentle grammars** that let $q$align **statistically** and **audibly**.
- We judge success not by absolute tracking error to a fixed template, but by **divergence that shrinks on average** while leaving room for expressive variation.

This is how the loop sounds **locked yet alive**: its certainty is inherited from the target; its life from the residual degrees of freedom.

7.2 Informational–Lyapunov sketch

We want a constructive way to talk about "keeping up with a moving truth." Let

$$V(t) := \; D\,(q(t) \;\|\; \tau(t))$$

be a **divergence** measuring how far the follower is from the driver. We keep D**copy-safe** and **contextual**:

- **Mechanical level (pre-music).** Let $q_m(t)$be a phase–amplitude descriptor of the pendulum (e.g., $(\phi, \dot{\phi})$relative to a driven orbit). Define

$$V_m(t) = \alpha\, \Delta\phi(t)^2 + \beta\, (\Delta A(t))^2,$$

where $\Delta\phi$is phase error to a target tongue and ΔAan amplitude mismatch.

- **Musical timing level (post-grammar).** Let $q_r(t)$be the realized **note process** (kick/hat times and velocities) and let $\tau_r(t)$describe the **implied grid** and density under current drive. Define

$$V_r(t) = \underbrace{\mathbb{E}\,|\,\Delta t_{\text{kick}}\,|}_{\text{foundation}} \;+\; \lambda\, \underbrace{\mathrm{Var}\,[\text{hat density} - \text{target density}]}_{\text{texture}},$$

or, equivalently, use cross-correlation deficits between hat activity and driver phase.

- **Timbre level.** Compare **CC 74** to a target envelope implied by energy flow (e.g., E_b):

$$V_c(t) = \| \;\mathrm{CC}_{74}(t) - \kappa\, \hat{E}_b(t)\; \|_{1,\text{per bar}}.$$

Design goal. Under **admissible drives** (bounded amplitude/frequency, optional dither) and **admissible feedback** (quantization windows, swing, refractory, density gates), we want

$$\frac{d}{dt}V(t) \;\leq\; 0 \text{on average over musical windows (beats→bars).}$$

Intuitively: each bar should **reduce or hold** divergence, even if single beats sometimes wander. This is *stability while truth moves*.

How to make $\dot{V} \leq 0$in practice (constructive levers)

- **Kind targets $\tau(t)$.**
    - Provide **capture plateaus** (Arnold tongues) via appropriate frequency and modest amplitude.
    - Add **HF dither** to **widen** the plateau (Kapitza kindness).
    - Use **chirps** (slow sweeps) for adiabatic on-ramps.

- **Gentle grammars (q rules).**
    - **Kick**: hard quantize to quarters with "guarantee fill" → clamps the foundation term in V_r.
    - **Hat**: swung 1/16 with snap windows → aligns statistically but preserves life.
    - **Crash**: bar-locked, refractory → avoids divergence spikes.
- **Selective feedback.**
    - **Refractory** and **Schmitt** logic prevent double-hits (sharp spikes in V_r).
    - **Density gates** and **probability thinning** hold texture variance inside bounds.
    - **CC smoothing** (10–20 Hz) ensures $V_c$doesn't oscillate at camera/ISR jitter rates.
- **Measurement loop.**
    - Log note times and CC values; compute V_r, V_cper bar; adjust swing, windows, Γ, chirp rate.
    - Seek monotone **barwise** decline or stationarity in Vduring "capture" and controlled rise during "release."

This is **not** a theorem; it's an **engineering discipline**: you shape the target and the coupling until the audible divergence behaves like a Lyapunov function.

7.3 Why the pendulum is a parable

The double pendulum incarnates the roles cleanly:

- **Base motion ≡** $\tau(t)$: the DAW-driven shaker (or simulated $g(t)$, $\phi(t)$) *proposes* a moving truth—tempo, phase, and gentle envelopes.
- **Lower link ≡** $q(t)$: the expressive tip *responds* with rich, nonlinear motion (captures, beats, hops). Through the **event grammar**, that motion becomes **kicks, hats, crashes, CCs**.
- **Music ≡ audible** $V(t)$:
    - When capture occurs, **kick timing error** and **hat density variance fall**—you *hear* $V \downarrow$as a **lock**.
    - During pump, CC tracks energy—V_c shrinks (brighter, steadier).

- During release, V **rises gracefully**—hats thin, CC exhales—yet the foundation remains intact because kick divergence is clamped by the grid.

This small theatre demonstrates the broader thesis: **coherence from resonance**. Instead of forcing the follower to match a static template, we **design the world's motion** to be easy to align with, and we **shape the coupling** so alignment is inevitable, audible, and kind. The result is not merely a working groove—it is a **didactic model** for any practice that seeks **order without rigidity** and **freedom without drift**.

8. Experiments You Can Actually Run

Each experiment is **doable tonight** with either the Python-only emulator or the bench-top rig. For every protocol you'll find: setup, procedure, measurements, analysis, success criteria, and failure fixes. Where "hardware" is mentioned, you can substitute the **emulator** by logging the same signals.

8.1 Timing bake-off: video-only vs. sensor ISR vs. hybrid (measure jitter)

Purpose. Quantify timing **jitter** and **latency** for drum-critical events across the three paths and relate those numbers to perceived tightness.

Setup.

- **Emulator or rig** producing kick ($\theta_2 \uparrow$ zero-cross) and hat ($|\theta_2| > r$).
- **DAW click** at 128 BPM recorded to an audio track; route detected **kick/hat** as MIDI to a **sampler** (short click samples with sharp transients).
- Optional: loop the MIDI out back into the DAW to print timestamps, or capture audio and detect peaks.

Procedure.

1. Run **video-only** (60 or 120 fps). Record 2–3 minutes of clicks (kick/hat).
2. Run **sensor ISR** (Leonardo interrupts). Record same length.
3. Run **hybrid** (kicks via ISR; CC via video). Record same length.

Measurements.

- For each event stream:

- Compute **beat-relative timing error** $e_i = t_i - \hat{t}_i$where $\hat{t}_i$is the nearest grid time after quantization.
- Report **RMS jitter** $\sqrt{\mathbb{E}[e_i^2]}$and **95% span** (P2.5–P97.5).
- Separate **pre-quantize** and **post-quantize** errors (use DAW's event log or emulator timestamps).

Analysis & success criteria.

- Expectations (post-quantize):
  - **Video-only:** Kick ≤ **15–25 ms** RMS, Hat ≤ **20–30 ms** RMS.
  - **Sensor ISR:** Kick ≤ **2–5 ms** RMS, Hat ≤ **5–10 ms** RMS.
  - **Hybrid:** Kicks match ISR; hats between ISR and video.
- **Subjective alignment:** listeners should rate ISR/hybrid kicks as "snappier" vs. video-only (A/B).

Failure fixes.

- High video jitter → lock exposure, go 120 fps, reduce snap window or increase swing.
- ISR double-hits → extend refractory (15–30 ms) or move sensors.

8.2 Drive sweeps: reveal Arnold tongues with audio-rate shakers

Purpose. Map **phase-lock plateaus** (Arnold tongues) by sweeping the base-drive frequency and measuring lock duration and quality.

Setup.

- Shaker puck driven by a sine from the DAW; amplitude fixed (choose Γ ≈ 0.08).
- Logger: phase error | $\theta_2 - n\theta_1$ |for $n \in \{1,2\}$OR musical proxies (hat density variance, crash rate).
- Emulation option: use the emulator's chirp_explore or manual sweep.

Procedure.

1. Sweep **frequency** upward: f_d: $0.6 \rightarrow 2.0$Hz over **4–8 minutes** (linear or logarithmic).
2. Then sweep **downward** (hysteresis check).

3. Repeat for **two amplitudes** (e.g., Γ = 0.06 and Γ = 0.12).

Measurements.

- **Lock indicator**: time segments where $|\ \theta_2 - 2\theta_1\ | < \varepsilon$ for ≥ 2 s (choose $\varepsilon = 0.10$ rad).
- **Plateau width**: Δf where lock persists.
- **Musical correlates**: decrease in hat-density variance; bar-locked crash frequency.

Analysis & success criteria.

- Expect distinct plateaus near **1:1** (direct) and **1:2** (parametric) relations.
- **Wider plateaus** at higher Γ; **hysteresis** (up vs. down sweep) indicates strong capture.

Failure fixes.

- No plateaus → reduce damping or raise amplitude slightly; add mild HF dither (see 8.3).
- Lock is noisy → increase snap windows, smooth CC.

8.3 HF dither tests: inverted-posture stabilization and groove stability

Purpose. Demonstrate **noise-assisted order**: tiny high-frequency drive increases stability (mechanically) and steadies the groove (musically).

Setup.

- Primary drive fixed near the **edge of lock** (found in 8.2).
- Add **HF dither**: sine at **10–20×** small-angle frequency, depth −18 to −24 dB relative to primary.

Procedure.

1. Alternate **2-minute blocks**: **OFF** (no dither) / **ON** (with dither) for 5 cycles.
2. Keep everything else constant.

Measurements.

- **Capture rate**: fraction of time locked per block.
- **Dropouts**: count per block.
- **Groove stability**: post-quantize kick jitter RMS; hat-density variance.

- Optional mechanical: average deviation from a near-inverted setpoint if attempting Kapitza stabilization in simulation.

Analysis & success criteria.

- With dither **ON**:
    - Lock fraction ↑ by ≥ **30%**, dropouts ↓.
    - Kick jitter stays low; hats fluctuate less erratically (variance ↓).
- Subjectively: “steadier,” “cleaner” groove without sounding sterile.

Failure fixes.

- Feels buzzy → reduce dither amplitude 3–6 dB.
- No effect → increase dither frequency (still >> musical band) or slightly raise depth.

8.4 Quantize sensitivity: feel vs. swing at 54–60%

Purpose. Find your **sweet-spot swing** and **snap windows** that maximize “alive” without mess.

Setup.

- Fixed drive (Γ ≈ 0.10), steady capture.
- Generate **five** 2-minute renders, identical except **swing** $s \in \{0.54, 0.56, 0.57, 0.58, 0.60\}$.
- For each swing, test **two** hat snap windows: **70 ms** and **90 ms**. (Total 10 conditions.)

Procedure.

1. Randomize playback order (A/B/X).
2. Ask 5–10 listeners (headphones) to score **tightness (1–7)** and **aliveness (1–7)**.
3. Collect objective metrics: hat jitter, density variance, CC variance.

Analysis & success criteria.

- Expect an **inverted-U** for “aliveness” peaking near **0.56–0.59**.
- Tightness tends to **decrease** with larger swing and larger snap.
- Choose the **Pareto** setting (max aliveness for acceptable tightness). Many land at **s=0.57, hat snap=80–90 ms**.

Failure fixes.

- If ratings cluster “stiff,” bump swing +0.02 and widen hat snap +10 ms.
- If “messy,” drop swing −0.02 and tighten snap −10 ms; consider minimum inter-hat 1/24 note.

8.5 Human-in-the-loop: map a knob to dither depth; assess perceived “life”

Purpose. Evaluate **performability**: can a single macro (HF-dither depth) act like a *“life fader”* without breaking the groove?

Setup.

- Map a **MIDI knob** (CC 21, say) to **HF dither depth** in the emulator or DAW (−∞ to −15 dB).
- Keep kicks/clocks unchanged; hats/CC as usual.

Procedure.

1. Perform **three 4-minute takes**:
    - **Static OFF** (no dither).
    - **Slow ride** (0 → −18 dB over 32 bars, then back).
    - **Dynamic ride** (musical swells: −24 ↔ −16 dB on builds/drops).
2. Record stereo output.
3. Blind-rate with 5–10 listeners: **engagement**, **fatigue**, **coherence** (1–7).

Measurements.

- Quantify **lock fraction**, **dropouts**, **CC variance**, and **hat density variance** per bar.
- Correlate knob position with those metrics.

Analysis & success criteria.

- **Slow ride** should increase engagement with minimal penalty to coherence.
- **Dynamic ride** should track perceived energy without introducing timing penalties (kick jitter stable).

Failure fixes.

- If the ride sounds noisy: low-pass the dither control or reduce the upper limit (cap at −18 dB).
- If nothing changes: ensure the dither frequency is sufficiently high and the main drive is near a tongue edge (where widening matters).

Logging templates (copy/paste)

- **Per-event CSV:** timestamp_sec, event_type(kick|hat|crash), pre_quant_err_ms, post_quant_err_ms, velocity
- **Per-bar CSV:** bar_idx, lock_fraction, kick_rms_ms, hat_rms_ms, hat_density, cc74_mean, cc74_std
- **Sweep CSV:** time_sec, fd_hz, Gamma, dither_db, phase_error_rad, locked(bool)

Decision table (quick use)

Question	Try First	If Too Stiff	If Too Messy
Tighten drums	ISR or Hybrid	—	—
More life	Swing 0.57, hat snap 80–90 ms	+swing +0.02	−swing −0.02; snap −10 ms
Longer locks	Add HF dither −20 dB	+3 dB dither	−3 dB dither; reduce Γ
Punctuated builds	Chirp into tongue	Slower α	Faster α; add lock-gap
Richer hats	Add r_2 gate (8.2)	Widen swing	Thin by probability or min-IOI

Run these, plot two figures (phase plateaus vs. frequency; listener ratings vs. swing), and you'll have **audible** and **numerical** evidence for the thesis: coherence emerges from **resonant alignment** to a kind, moving **τ(t)**.

9. Predictions, Metrics, Falsifiability

This section states **testable predictions**, the **metrics** that adjudicate them, and **clear falsification criteria**. Wherever possible, we propose small-N lab protocols that scale to open, remote replication.

9.1 Listeners rate hybrid grooves as "more alive" than grid-only or video-only (blind A/B/X)

Prediction. Under identical musical content, **Hybrid** (kicks via ISR, hats via sensors, CC via video) will be rated **higher on "aliveness"** than **Grid-only** (hand-quantized drum pattern + static automation) and **Video-only** (camera timing for all events, quantized).

Stimuli.

- Three 2-minute loops per condition, **128 BPM**, same drum kit and synth.
- Hybrid: our default mapping (§2, §3.3).
- Video-only: hats & kicks detected from video; quantized as in §2.4; same CC path.
- Grid-only: kicks quarter-grid, hats swung 1/16 at s=0.57; **no CC** or **static CC** (counterbalanced).

Participants. 16–30 listeners with headphone monitoring; no training required.

Task. A/B/X trials: "Which is **more alive** yet still **tight**?" (two Likerts: Aliveness 1–7, Tightness 1–7). Randomized order; at least **12 trials** per participant.

Primary metrics.

- Mean **Aliveness**: $\bar{A}_{\text{Hybrid}} - \bar{A}_{\text{Grid}}$, $\bar{A}_{\text{Hybrid}} - \bar{A}_{\text{Video}}$.
- Secondary: **Tightness** differences; choice proportion in **X** trials.

Analysis.

- **Within-subject** repeated-measures ANOVA (or linear mixed effects) on Aliveness; report **η^2** or **Cohen's d**.
- **Power hint:** expecting $d \approx 0.5$, $n = 24$gives ~80% power (paired t).

Falsification.

- If Hybrid's mean Aliveness ≤ Grid-only by CI overlap and **p ≥ 0.05** (two-sided), or Hybrid underperforms Video-only, **prediction fails**.
- We will publish raw MIDI, audio bounces, and per-trial CSVs regardless of outcome.

9.2 Groove stability widens with modest HF dither without perceived harshness

Prediction. Adding **HF dither** (−18 to −24 dB at 10–20× ω_p) **increases lock fraction** and **reduces dropouts** without decreasing listener **Pleasantness**.

Protocol. OFF/ON alternating blocks (5×2 minutes each) at the edge of a phase-lock tongue (§8.3).

Metrics.

- **Lock fraction** per block: proportion of time | $\theta_2 - 2\theta_1$ |$< \varepsilon$for ≥ 2 s.
- **Dropouts/hour**: count of exits from lock.
- **Kick RMS jitter** (post-quantize).
- **Pleasantness** (Likert 1–7) from blind A/B of matched OFF vs ON audio.

Analysis.

- Paired comparisons OFF vs ON; report **ΔLock**, **ΔDropouts**, **ΔJitter**, **ΔPleasantness**.
- Equivalence test on Pleasantness to assert **no degradation** (TOST with Δ=0.5 Likert).

Falsification.

- If **ΔLock ≤ 0** and **ΔDropouts ≥ 0** or Pleasantness decreases (p<.05), **prediction fails**.

9.3 Near-1:2 lock events correlate with preferred "crash" placement across sets

Prediction. Crash onsets (after quantizing to bar downbeats) that coincide with **near-1:2** mechanical relations are **preferred** (rated more satisfying) vs. time-matched controls.

Stimuli.

- 24 bars per take; 6 crash candidates per take.
- Two conditions per candidate: **Lock-aligned** (= event when | $\theta_2 - 2\theta_1$ |$< \varepsilon$), **Control** (= same bar index from a different take without near-lock).

Metrics.

- **Preference rate** (% trials Lock-aligned chosen over Control).
- **Satisfaction score** (Likert 1–7).
- **Mechanical strength** of alignment (mean | $\theta_2 - 2\theta_1$ |in the −400…0 ms window).

Analysis.

- **Logistic regression**: Choice ~ AlignmentStrength + subject random intercept.
- **Correlation**: Satisfaction vs AlignmentStrength (Spearman).

Falsification.

- If Preference ≤ 50% with CI including 50% and no correlation (p≥.05), **prediction fails**.

9.4 CC-driven filters outperform static automation in perceived non-fatigue over 5–10-minute loops

Prediction. Loops with **CC 74** from motion speed/energy (§2.3) are rated **less fatiguing** and **more engaging** than **static** or **random-walk** automation matched for mean/variance.

Stimuli.

- Three 8-minute bounces from the same MIDI drums:
    1. **Motion-CC** (sampled each swung 1/16).
    2. **Static** (fixed cutoff at Motion-CC mean).
    3. **Random-walk CC** (matched variance & spectrum; no motion coupling).

Task.

- Continuous **slider** (0–100) for **Engagement**; post-block **Fatigue** (1–7) and **Willingness-to-keep-listening** (binary).

Metrics.

- Mean Engagement; slope over time (does it decay?); Fatigue score; survival curve for willingness.

Analysis.

- Mixed models for Engagement ~ Condition × Time; Kaplan–Meier for willingness.
- Expect **shallower decay** and **lower fatigue** for Motion-CC.

Falsification.

- If Motion-CC does not improve Engagement slope or reduces Fatigue vs Static/Random (p<.05), **prediction fails**.

9.5 Open-source replication: MIDI files and parameter logs match claims

Prediction. Independent users running the provided **emulator** (Appendix E) with the specified flags reproduce: (i) event densities, (ii) timing statistics, (iii) lock fractions **within tolerance**.

Release artifacts.

- **Exact CLI** used (e.g., --preset trance128 --minutes 3 --swing 0.57 --gamma 0.15 --fd 1.2).
- **Seeds & versions**: Python version, commit hash, OS.
- **Outputs**: .mid files, **per-event CSV**, **per-bar CSV**, consolidated **metrics.json**.

Tolerance bands (pass/fail).

- Kick count per minute: ±2%.
- Hat count per minute: ±5%.
- Post-quantize jitter (RMS): ±3 ms absolute.
- Lock fraction (over 3 minutes): ±0.05 absolute.

Replication checklist.

1. Run CLI, produce files.
2. Verify hashes of .mid and metrics.json (or numeric closeness where floating-point platforms differ).
3. Submit a short **replication report** (auto-generated notebook) to the repo.

Falsification.

- If ≥20% of independent reports fall **outside tolerance** (with environment info provided), **prediction fails** (or tooling is at fault—must fix and re-test).

Shared Measurement Definitions (to avoid garden-paths)

- **Kick jitter (post-quantize).** RMS of $t_i - \hat{t}_i$over all kicks within a bar, averaged across bars.
- **Hat density variance.** Variance of hat counts per bar normalized by mean (Fano factor).
- **Lock fraction.** Time proportion where $|\ \theta_2 - 2\theta_1\ | < \varepsilon$for ≥2 s, with $\varepsilon = 0.10\ \mathrm{r}\qquad\qquad\qquad\qquad\qquad\qquad\mathrm{ad}.$
- **Alignment strength (crash).** Mean of $1 - \frac{|\theta_2 - 2\theta_1|}{\varepsilon}$clipped to [0,1] over −400…0 ms pre-crash.

Preregistration Template (minimal)

- **Hypotheses:** 9.1–9.4 as above.
- **Primary endpoints:** Aliveness (9.1), Lock fraction (9.2), Preference rate (9.3), Engagement slope & Fatigue (9.4).
- **Analysis plan:** tests specified per subsection; α = 0.05, two-sided; corrections via Holm–Bonferroni across primary endpoints.
- **Stopping rule:** fixed N per protocol (≥24 listeners for 9.1, ≥16 for 9.3–9.4).
- **Exclusions:** headphone noncompliance (pre-test fails), inattentive responders (catch trials <75% correct), audio clipping.

Data & Code to Publish

- **Code:** simulator, MIDI writer, loggers, analysis notebooks.
- **Data:** all .mid, .wav bounces, per-event and per-bar CSVs, anonymized listener CSVs.
- **Figures:**
    - Arnold-tongue map (Δf vs Γ).
    - Engagement traces (per condition).
    - Preference forest plots (9.3).
    - Jitter boxplots (8.1).

Bottom line

Each prediction is **audible**, **measurable**, and **falsifiable**. If the numbers don't move the way we say—or if independent labs can't reproduce them—the thesis is wrong in that slice, and the paper has done its job by showing **where** resonance fails to beat coercion.

10. Beyond the Studio: Macro Thought Experiments

We extend **q → τ(t)** past music. In each domain, we (a) propose **what τ(t) is**, (b) design **couplings** so followers $q$resonate rather than obey, (c) suggest **metrics** (audible/visible proxies for $V(t) = D(q \parallel \tau(t))$), and (d) name **failure modes** plus **kindness levers** (how to make τ(t) generous).

10.1 Buildings that entrain occupants (acoustics, daylight, micro-motions)

τ(t). A building's **temporal affordances**: daylight cycles, air pulses, acoustic decays, micro-motions in furniture/floor (imperceptible sway), escalator/headway rhythms, HVAC breath.

Couplings (how people "hear" τ).

- **Light:** circadian-friendly **CCT drift** (e.g., 3100→4200 K across late morning; asymmetrical warm-down at dusk) synchronized to local solar τ(t); micro-flicker avoided.
- **Acoustics:** rooms tuned for **repeatable RT60** and **low flutter**, plus subtle **soundmarks** (e.g., water walls, white/pink masks) with **slow amplitude envelopes**—breathing spaces that reduce cognitive V.
- **Micro-motion:** *inaudible* floor/furniture sway on **very low frequency** (0.1–0.3 Hz) within building code—perceptual effect is a **settling** rhythm, not seasickness; strictly opt-in zones.
- **Air:** HVAC modulation with **broadband low-variance** flow; minute-scale **"open–rest"** patterns that align to work sprints.

Metrics (copy-safe V proxies).

- **Attention stability:** variance of head/eye micro-saccade rates, task-switching frequency; self-reported **calm/flow** (Likert).
- **Physiology:** HRV (RMSSD) during work blocks; light dose (lux×time at eye).
- **Acoustics:** STI, RT60 drift over day (should be stable); spectrum of background—no harsh peaks.

Kindness levers.

- **Personal control** (local dimmers, task lights).
- **Soft ramps** (fade > 20 s for light/air changes).

- **Choice architecture:** “quiet tongues” (libraries), “buzz tongues” (cafés) so users self-select τ(t).

Failure modes.

- Over-regularity → **boredom** (over-lock).
- Over-complexity → **fatigue** (chaotic τ).
- **Ethical red line:** no covert behavior nudging; disclose rhythmic designs; opt-out always.

10.2 Instruments that play with fidelity (tools whose bases are τ(t)-aware)

τ(t). Tool or instrument **base motion / timing** that moves first: metronomic micro-actuation, adaptive latency compensation, haptic breath.

Couplings.

- **Haptic τ:** controllers with **subtle force fields** (0.2–0.5 N) that **guide** toward groove arcs without locking the hand; viscosity increases near intended capture zones (like widened Arnold tongues).
- **Audio τ:** click-tracks that are **not clicks**—low, musical pulsations with **swinged subdivisions**; **tempo-chirp** intros for adiabatic entry into tight playing.
- **Latency τ:** pedals and live FX that measure player onset lag and **shift buffers** to keep IOI stable; τ(t) surfaces as **predictable feel**.

Metrics.

- **Timing error** (RMS to intended grid), **timing drift** (slope), **microdynamic variance** preserved vs. a hard quantizer baseline.
- **User effort** (EMG, grip force), **learning curves** (trials to criterion).
- **Preference** in blind A/B for “plays like butter.”

Kindness levers.

- **Adaptive swing windows** that grow or shrink with player consistency.
- **Sympathetic CC** (filter/pan) that mirrors player speed—audible $V \downarrow$.
- **Transparent UI**: display where τ(t) is (e.g., moving bead).

Failure modes.

- Overhelp → **rubber-band feel** (agency loss).
- Underhelp → **jittery**, discouraging.
- **Ethics:** expose assistance level; record both **raw** and **assisted** streams.

10.3 Cities as resonance engines (transit/headways as base motion)

τ(t). Transit headways, pedestrian crossing phases, elevator dispatch, plaza soundmarks—city rhythms that residents can phase-lock to without coercion.

Couplings.

- **Headway τ:** buses/trains aim for **regular headways** (not only schedule), using **gentle holding** upstream to dissolve platoons—passengers inherit a **low-divergence** waiting experience.
- **Crossing τ:** pedestrian signals with **predictable green waves** and **countdown chirps** that are **honest** (no false urgency); speeds align; conflict lowers.
- **Plaza τ:** fountains/lights with **minute-scale envelopes**, aligning social groups; spatialized sound that **breathes** with occupancy (sensors adjust **slowly**, no jump scares).
- **Bike τ:** subtle **lead-in ramps** at intersections (slight grade or paint cues) to regularize starts.

Metrics.

- **Headway variance** (coefficient of variation), **wait-time Gini**, **missed-connection rate**.
- **Conflict metrics** at crossings (near-misses), **speed dispersion** histograms.
- **Dwell time** in plazas, **vibe** ratings in intercept surveys.

Kindness levers.

- **Slack** in schedules to allow holding; **soft information** (accurate arrival times).
- **Monotone adjustments** (smooth frequency-domain changes, not jerks).
- **Participatory mapping**: publish τ(t) so neighborhoods can propose better couplings.

Failure modes.

- Over-optimization → brittle to shocks (special events, weather).

- Hidden coercion (e.g., “hostile” tempos to displace loiterers)—**reject**; use inclusive design.

10.4 Liturgy and pedagogy (ritual clocks as gentle τ(t) to align attention)

τ(t). Ritual clocks—lesson pacing, recitations, call-and-response, breathwork, silences; in faith settings, canonical hours, chant tempos, procession steps.

Couplings.

- **Pedagogy:** lesson arcs with **adiabatic ramps** (settle → engage → drill → release). Use **call-and-response** timing that students can **predict**; include **swinged micro-pauses** to signal turn-taking.
- **Liturgy:** chants with **breath τ** (measured phrases, common cadence); **silence windows** that are **long enough** to settle but bounded; processional tempos that **fit stride** diversity.

Metrics.

- **Attention drift** (teacher gaze tracking; response latency in prompts).
- **Recall/transfer** scores; **affect** (self-report calm/engagement).
- **Congregational coherence**: onset spread in communal responses, breathing synchrony (safe proxies).

Kindness levers.

- **Multiple capture plateaus**: alternative tempos for accessibility; **call-out cues** before transitions.
- **Non-coercive** invitations: “you may join on the next phrase.”
- **Feedback**: leaders adjust τ(t) by reading group V(hesitation, rush).

Failure modes.

- Over-rigid ceremony → **rote compliance** (over-lock; low learning/meaning).
- Over-fluid pacing → **drift** (no shared structure).
- **Ethics:** zero manipulation; disclose aims (attention, calm); accommodate neurodiversity and mobility differences.

Synthesis (one paragraph)

Across domains, the **design pattern** is the same: choose a **kind target** τ(t) with **capture plateaus** and **soft on-ramps**, expose it **honestly**, couple followers $q$with **forgiving grammars**, and watch $V(t) = D(q \parallel \tau(t))$**fall on average**—without freezing out life. Whether it's a loop, a room, a violin, a bus line, or a classroom, **resonance beats coercion** when we make the world's motion **audible, navigable, and kind**.

11. Ethics and Limits

Designing with **q → τ(t)** is powerful precisely because people and systems so readily entrain. That power comes with responsibility. This section names the boundary conditions: how to **invite** resonance without **imposing** it; how to keep users **informed and consenting**; and how to avoid the practical harms that arise when targets are too rigid, too chaotic, or too strong.

11.1 Resonance vs. control: designing without coercion

Stance. Make the world's motion $\tau(t)$**legible, gentle, and optional**. The aim is **alignment by attraction**, not adherence by force.

Design principles

- **Kind targets.** Favor capture plateaus with **soft on-ramps** (e.g., slow chirps, mild HF dither) over hard snaps. If a user walks away, the system should **gracefully drift** rather than punish.
- **Preserve agency.** Keep a **degree of freedom** for the follower: tight kicks + living hats/CCs is not just a musical trick—it is an ethical pattern (certainty + choice).
- **Local override.** Always expose **manual controls** (bypass quantize, mute controllers, pause drives). A system that cannot be paused is not a partner; it is a boss.
- **Proportionality.** Match the "pull" of $\tau(t)$to context. Studio experiment? You may dial Γ higher. Public or shared space? **Lower amplitudes, slower ramps**, wider opt-out.
- **Explainability.** If a behavior emerges, you should be able to point to **specific couplings** (e.g., "this crash because near-1:2 lock") rather than appeal to mystique.

Red flags (coercion smells)

- Hidden metronomes or haptics that bias timing **without disclosure**.

- Removing bypass/stop under the guise of “stability.”
- Targets tuned to exploit fatigue or attention gaps (e.g., “infinite drop” pacing).

11.2 Consent and transparency: when groove becomes manipulation

Rule-of-thumb. When rhythmic or timbral design **predictably shapes behavior** (linger time, purchase, compliance), it crosses from “aesthetic” to “behavioral”—that requires **explicit consent**.

Transparency checklist

- **What:** Name the driver $\tau(t)$: “This platform uses a low-frequency base motion synced to the audio clock.”
- **Why:** State intent: “to stabilize timing and reduce jitter,” not “to keep you here longer.”
- **How:** Describe coupling in copy-safe terms (swing %, snap windows, dither depth).
- **Control:** Provide toggles/knobs with sensible defaults and **no re-enable dark patterns**.

Consent patterns

- **Opt-in** for nonessential entrainment (e.g., haptic guidance, attention pacing).
- **Session-scoped** permission with clear **expiry** (“for this set only”).
- **Granular choices**: e.g., “tighten drums” yes; “modulate my breathing” no.

When to pull back

- If participants report **compulsion** (“I felt pulled along”) without understanding why.
- If metrics show **asymmetric effects** on vulnerable groups (motion sensitivity, neurodiversity).
- If the design’s purpose is **not aligned** with the user’s goals (e.g., covertly boosting dwell time).

Disclosure language (plain)

“We use gentle, clock-true motion to keep timing steady and let textures breathe. You can turn this off anytime. Here’s what will change if you do.”

11.3 Failure modes: boredom (over-lock), nausea (over-chaos), injury (over-drive)

Even with good intent, dynamics can go wrong. Treat these as **engineering hazards** with **mitigations**.

A) Boredom (over-lock)

Symptoms

- Loop feels **sterile**, dynamics flatten, listeners disengage.
- Metrics: hat density variance → **near zero**; CC variance decays over minutes.

Causes

- Excessively strong or narrow targets (large Γ, no chirp/dither); swing too low; snap windows too tight.

Mitigations

- **Loosen**: raise swing (e.g., 0.57→0.59), widen hat snap (70→90 ms), add **micro-dither** or slow **two-tone** bias.
- **Alternate tongues**: brief chirps into/out of 1:1 and 1:2 zones.
- **Texture rotation**: map a second gate (different radius) to share density.

B) Nausea / fatigue (over-chaos)

Symptoms

- Perceived **jitter**, "washy" pulse, sensory overload; motion sickness in physical rigs.
- Metrics: post-quantize jitter ↑; near-lock detector thrashes; CC flicker > 20 Hz.

Causes

- Drive too hot; damping too low; video latency/jitter leaking into drums; CC unfiltered.

Mitigations

- **Clamp foundation**: hard quantize kicks; raise damping slightly; reduce Γ.
- **Filter**: low-pass CC at 10–20 Hz; add Schmitt + refractory to hats.
- **Slow down**: prefer **adiabatic** sweeps; avoid rapid parameter toggles.

C) Injury / damage (over-drive)

Symptoms

- Hardware: loose fasteners, cracked mounts, overheated coils/amps.
- Human: pinches, burns, hearing fatigue.

Causes

- Excess amplitude/frequency; unguarded moving parts; missing flyback diodes; clipping amplifiers; prolonged high-SPL monitoring.

Mitigations

- **Safety envelopes**: base motion 1–5 mm starter, HF dither <1 mm; incremental changes.
- **Physical guards**: soft end-stops, finger-safe clearances, strain relief, threadlocker.
- **Electrical hygiene**: flyback diodes on coils; separate grounds; fuses; thermal checks.
- **Hearing care**: monitor at reasonable SPL; schedule breaks.

Ethical operating procedure (one-page)

- **Declare τ(t)** (what/why/how).
- **Offer control** (bypass, per-channel quantize, dither depth).
- **Log & review** divergence proxies (bar-wise V for timing, density, CC).
- **Watch for red flags** (compulsion reports, asymmetric effects).
- **Fail safe** (overrides, soft limits, physical guards).
- **Publish** enough detail for critique (settings, code, envelopes).

Bottom line. If users can **see the target**, **choose the coupling**, and **leave without penalty**, you are designing for **resonance**. If they cannot, you are courting **control**—and the system, however clever, has missed the point of **q → τ(t)**.

12. Implementation Appendix (practical)

Below is a **one-stop, buildable pack**: a cart you can assemble from commodity parts, a drop-in **Leonardo USB-MIDI** sketch for real-time drums, a **Python video tracker** outline, the **simulation→MIDI** path, and **safety envelopes** that keep rigs—and people—out of trouble.

A. Amazon-only bill of materials (representative, returnable SKUs)

Swap brands freely—prioritize the *specs* in bold. One of each unless noted.

Motion & Power

- **Mini tactile transducer ("shaker puck")** — **4–8 Ω**, **10–50 W**, Ø 35–50 mm, adhesive mount.
- **Class-D amplifier** — **1×50 W** (or 2×50 W; use one channel), **RCA/3.5 mm line-in**, 12–24 V DC input.
- **Laptop-style DC supply** — **24 V / 3–5 A** (or per amp spec), UL-listed.
- *(Optional)* **Rotisserie/BBQ motor** — **DC 12–24 V** preferred (speed tweakable), **≥10 kg·cm torque**, square-shaft coupler.
- *(Optional)* **Eccentric crank kit** — small arm with slot, M3/M4 hardware.

Sensing & Control

- **Arduino Leonardo** (ATmega32u4, **native USB-MIDI**).
- **Hall effect switch** (digital, omnipolar or latching) + **3–5 mm neodymium magnet** (on lower bob).
- **Photointerrupter** (slot-type) *or* **reflective IR sensor** module.
- **Jumper wires**, Dupont leads, **10 kΩ** resistors (pull-ups), **100 Ω** gate resistor (if you add a MOSFET).
- *(Optional)* **12 V solenoid** (5–10 mm stroke) + **cycle timer** (adjustable on/off), **flyback diode** (1N5819/UF4007).
- **USB cable** with **ferrite** choke.

Mounting & Misc

- **Base plate** (plywood or 3–5 mm aluminum, ~20–30 cm square) + **rubber feet** / sorbothane sheet.
- **C-clamps/quick clamps**, **threadlocker (blue)**, **VHB tape / epoxy**, **zip ties** + adhesive anchors, **foam bumpers**.
- *(Optional)* **Small tripod & USB camera** (60–120 fps).

Quick start: if you must pick only a few items—**puck + amp + Leonardo + Hall + photogate + clamps**—you can run the complete hybrid protocol.

B. Arduino Leonardo USB-MIDI sketch (Hall + photogate → notes/CC)

What it does

- Hall centerline crossing ⇒ **Kick (Note 36)** on CH10, velocity from time-since-last crossing.
- Photogate mid-radius entry ⇒ **Hat (Note 42)** on CH10, velocity from amplitude proxy.
- Emits **CC 74** on CH2 every 1/16 (derived from inter-edge speeds), range **10–120**.
- Debounced with **Schmitt logic** + **refractory**; jitter-tight via **interrupts**.

Libraries: install **MIDIUSB** (Arduino Library Manager) or use the built-in "USB MIDI" core if your IDE offers it. The sketch below uses MIDIUSB.h.

```
/* Leonardo USB-MIDI: Hall→Kick, Photogate→Hat, CC74 from speed.
   Pins: D2 (INT0) = Hall, D3 (INT1) = Photogate.
   Set photogate active level to match your module (LOW or HIGH).
*/
#include <MIDIUSB.h>

const uint8_t CH_DRUM = 9;   // Channel 10 (0-based)
const uint8_t CH_SYN  = 1;   // Channel 2 (0-based)
const uint8_t KICK = 36, HAT = 42, CRASH = 49;

const uint8_t PIN_HALL = 2;      // INT0
const uint8_t PIN_GATE = 3;      // INT1
const bool    GATE_ACTIVE_HIGH = true;

volatile uint32_t lastKickMs = 0, lastHatMs = 0;
volatile bool hallEdge = false, gateEdge = false;

const uint16_t REFRACT_KICK_MS = 30;
```

```
const uint16_t REFRACT_HAT_MS  = 25;
const uint16_t CC_PERIOD_MS    =  (60000 / 128) / 4; // 1/16 at 128 BPM ≈ 117ms
uint32_t lastCcMs = 0;

// Schmitt emulation for photogate
volatile bool gateState = false;

void noteOn(uint8_t ch, uint8_t note, uint8_t vel) {
  midiEventPacket_t p = {0x09, uint8_t(0x90 | ch), note, vel};
  MidiUSB.sendMIDI(p);
}
void noteOff(uint8_t ch, uint8_t note) {
  midiEventPacket_t p = {0x08, uint8_t(0x80 | ch), note, 0};
  MidiUSB.sendMIDI(p);
}
void cc(uint8_t ch, uint8_t num, uint8_t val) {
  midiEventPacket_t p = {0x0B, uint8_t(0xB0 | ch), num, val};
  MidiUSB.sendMIDI(p);
}
void flush() { MidiUSB.flush(); }

void isr_hall() { hallEdge = true; }
void isr_gate() {
  bool lv = digitalRead(PIN_GATE);
  bool active = GATE_ACTIVE_HIGH ? lv : !lv;
  if (active && !gateState) { gateEdge = true; }
```

```
  gateState = active;
}

void setup() {
  pinMode(PIN_HALL, INPUT_PULLUP);
  pinMode(PIN_GATE, GATE_ACTIVE_HIGH ? INPUT : INPUT_PULLUP);
  attachInterrupt(digitalPinToInterrupt(PIN_HALL), isr_hall, CHANGE);
  attachInterrupt(digitalPinToInterrupt(PIN_GATE), isr_gate, CHANGE);
}

uint8_t clamp(uint8_t x, uint8_t a, uint8_t b){ return x<a? a : (x>b? b : x); }

void loop() {
  uint32_t now = millis();

  // KICK from Hall upward crossing — treat any edge with refractory as acceptable proxy
  if (hallEdge) {
    hallEdge = false;
    if (now - lastKickMs >= REFRACT_KICK_MS) {
      uint32_t dt = now - lastKickMs;
      lastKickMs = now;
      // Velocity from "speed": smaller dt ⇒ higher vel (clip)
      uint8_t vel = clamp(30 + (uint16_t) (90.0 * (1.0 - min(1.0, dt / 250.0))), 30, 127);
      noteOn(CH_DRUM, KICK, vel);
      // short note
      delay(5);
```

```
    noteOff(CH_DRUM, KICK);
    flush();
  }
}

// HAT from photogate "enter"
if (gateEdge) {
  gateEdge = false;
  if (now - lastHatMs >= REFRACT_HAT_MS) {
    lastHatMs = now;
    // Simple amplitude proxy: time since last hat; longer gap = larger swing
    uint32_t dt = now - lastHatMs; // NOTE: lastHatMs just set; use a separate state if preferred
    uint8_t vel = clamp(40 + (uint16_t)(70.0 * 0.7 /*ampl proxy*/ ), 30, 127);
    noteOn(CH_DRUM, HAT, vel);
    delay(3);
    noteOff(CH_DRUM, HAT);
    flush();
  }
}

// CC74 every 1/16
if (now - lastCcMs >= CC_PERIOD_MS) {
  lastCcMs = now;
  // Crude speed proxy: mix of recent inter-kick/hat times; here we just dither softly
  static uint8_t ccv = 64;
  // In practice compute from ISR-derived speeds; placeholder gentle wander:
```

```
    int delta = (int)(random(-2, 3));
    int val = (int)ccv + delta;
    ccv = clamp(val, 10, 120);
    cc(CH_SYN, 74, ccv);
    flush();
  }
}
```

Wiring (text)

- **Hall**: Vcc 5 V → Hall V+, GND → GND, OUT → **D2** (INT0). If open-collector, add 10 kΩ pull-up to 5 V.
- **Photogate**: Vcc 5 V → V+, GND → GND, OUT → **D3** (INT1). Set GATE_ACTIVE_HIGH to match module.
- **Shaker puck**: amp CH-A → puck; amp line-in from audio interface output bus.

For production, compute CC from true inter-edge speeds (keep a short circular buffer of last N ISR timestamps), and (optionally) send **crashes** on bar downbeats by counting kicks.

C. Python video tracker outline (OpenCV thresholds, zero-cross logic, OSC/MIDI out)

Purpose
A minimal camera-only path: detect the bob, compute $\theta_2(t)$, trigger events, and send MIDI/OSC to your DAW.

Dependencies
opencv-python, numpy, python-rtmidi *(or)* mido + python-rtmidi, optionally python-osc.

Pipeline outline (well-commented skeleton)

```
# video_trig.py — Camera→Events→MIDI
import cv2, numpy as np, time, math
from mido import Message, MidiFile, MidiTrack, MetaMessage, open_output
```

```
# ---- Camera / ROI
cap = cv2.VideoCapture(0)
cap.set(cv2.CAP_PROP_FPS, 120)       # aim high; 60 also fine
cap.set(cv2.CAP_PROP_FRAME_WIDTH, 1280)
cap.set(cv2.CAP_PROP_FRAME_HEIGHT, 720)

# ---- MIDI out
out = open_output()  # default virtual port

# ---- Params
BPM = 128
r_low, r_high = 0.32, 0.38          # Schmitt band for hats (radians, approx)
swing = 0.57
refract_kick_ms = 30
refract_hat_ms  = 25

def grid_times(bpm, seconds, swing):
    # return swung 1/16 times and quarters (sec) for quantization
    s_per_q = 60.0 / bpm
    s_per_16 = s_per_q / 4.0
    t, g16, qtrs = 0.0, [], []
    while t < seconds + 1e-6:
        base = int(t / s_per_q) * s_per_q
        for k in range(4):
            straight = base + k*s_per_16
            if k % 2:
```

```
            prev = base + (k-1)*s_per_16
            nxt  = base + (k+1)*s_per_16 if k<3 else base + s_per_q
            g16.append(prev + swing*(nxt - prev))
        else:
            g16.append(straight)
    qtrs.append(base)
    t = base + s_per_q
  # Dedup/sort
  g16 = sorted(set(round(x, 3) for x in g16))
  qtrs = sorted(set(round(x, 3) for x in qtrs))
  return g16, qtrs

grid16, quarters = grid_times(BPM, 3600, swing)  # 1 hr buffers

def quantize(ts, grid, snap=0.08):              # snap in seconds
  idx = np.searchsorted(grid, ts)
  cands = [grid[max(0, idx-1)]]
  if idx < len(grid): cands.append(grid[idx])
  q = min(cands, key=lambda g: abs(g-ts))
  return q if abs(q-ts) <= snap else None

# ---- Helpers
def midi_note(ch, note, vel, on=True):
  out.send(Message('note_on' if on else 'note_off', channel=ch, note=note, velocity=vel))

def midi_cc(ch, ccnum, val):
```

```
    out.send(Message('control_change', channel=ch, control=ccnum, value=val))

# ---- State
last_kick_t, last_hat_t = 0, 0
prev_theta2 = None
above = False

# ---- Main loop
t0 = time.time()
while True:
    ok, frame = cap.read()
    if not ok: break
    ts = time.time() - t0

    # 1) find bob (HSV threshold or ArUco; here: color blob)
    hsv = cv2.cvtColor(frame, cv2.COLOR_BGR2HSV)
    mask = cv2.inRange(hsv, (20, 80, 80), (35, 255, 255))   # tune color!
    cnts, _ = cv2.findContours(mask, cv2.RETR_EXTERNAL, cv2.CHAIN_APPROX_SIMPLE)
    if not cnts: continue
    c = max(cnts, key=cv2.contourArea)
    (x, y), r = cv2.minEnclosingCircle(c)
    if r < 3: continue

    # 2) map (x,y) to theta2 (calibrate origin/scale for your rig)
    cx, cy = frame.shape[1]//2, 100  # example: pivot above
    theta2 = math.atan2(y - cy, x - cx)  # crude; refine in practice
```

```
# 3) events
# Kick: upward zero-cross
if prev_theta2 is not None and prev_theta2 < 0 and theta2 >= 0:
    if (ts - last_kick_t) * 1000 >= refract_kick_ms:
        q = quantize(ts, quarters, snap=0.20)
        if q is not None:
            midi_note(9, 36, 96, True); midi_note(9, 36, 0, False)
            last_kick_t = ts

# Hat: enter |theta2| > r_high
was_above = above
above = abs(theta2) > r_high if not was_above else abs(theta2) > r_low
if (not was_above) and above:
    if (ts - last_hat_t) * 1000 >= refract_hat_ms:
        q = quantize(ts, grid16, snap=0.08)
        if q is not None:
            vel = 40 + int(70 * min(1.0, abs(theta2) / (math.pi/2)))
            midi_note(9, 42, max(30, min(127, vel)), True)
            midi_note(9, 42, 0, False)
            last_hat_t = ts

# CC 74 each 1/16 (naïve sampling here: every N frames)
if int(ts * (BPM/60*4)) != int((ts-1e-3) * (BPM/60*4)):
    speed = 0.5  # TODO: estimate via finite differences + smoothing
    ccval = 10 + int(110 * max(0.0, min(1.0, speed)))
```

```
    midi_cc(1, 74, ccval)


prev_theta2 = theta2


# (optional) visualize
# cv2.circle(frame, (int(x),int(y)), int(r), (0,255,0), 2)
# cv2.imshow('bob', frame);  cv2.waitKey(1)
```

Tuning notes

- Lock camera **exposure/gain**; avoid PWM/flicker lighting.
- Calibrate pivot point (cx, cy) and the radians/px scale from a static pose.
- Replace the placeholder **speed** with a smoothed derivative of theta2.

D. Simulation→MIDI generator (event grammar, quantizer, SMF writer)

You already have the **single-file emulator** earlier in this document (Appendix E in our previous section). Here is the **operator's card** + **core API map** so you can extend it:

CLI (ready to run)

```
python pendulum_emulator.py --preset trance128 --minutes 3 --out trance_128.mid

# Variations:

python pendulum_emulator.py --preset raw_chaos --minutes 2 --swing 0.58 --out chaos.mid

python pendulum_emulator.py --preset chirp_explore --minutes 4 --fd 0.8 --chirp-rate 0.05 --out chirp.mid
```

Config levers (essentials)

- **Tempo/grid**: --bpm 128 --ppq 480 --swing 0.57
- **Integrator**: --dt 0.0025 (≈400 Hz)
- **Drive**: --preset {trance128|soft_ambient|chirp_explore|raw_chaos} *or* --drive {parametric|tilt|quasiperiodic|chirp|kicks} with --gamma --fd --fd2 --chirp-rate
- **Detection**: --hat-th 0.35 --lock-eps 0.10 --lock-gap 2.0

Internals (one-screen map)

- DoublePendulum.rk4(t, dt, drive) → integrates $(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2)$with **parametric g(t)** and optional **tilt**.
- detect_and_simulate(cfg, drive) →
    - **Kick** at $\theta_2 \uparrow 0$; **Hat** at $|\ \theta_2\ | > r$(enter); **Crash** near $|\ \theta_2 - 2\theta_1\ | < \varepsilon$.
    - Returns event time lists + **speed series** for CC.
- build_swung_grid(T,bpm,ppq,swing) → swung 1/16 & quarter grids (sec).
- quantize(events, grid, snap_window) → snap with de-dupe & windowing.
- build_midi(...) → **SMF Type-1**: CH10 (36/42/49), CH2 CC74; meta: tempo & 4/4.

Integration tests (sanity)

- **Bit-identical** output for same params on same platform.
- **Events** present: kicks ≥ bars, hats hundreds/min, crashes sparse.
- **SMF** loads in DAW; CC74 visible on CH2.

E. Safety envelopes (amplitudes, frequencies, guarding, electrical protection)

Motion envelopes (start here, then creep up)

- **Base motion amplitude (puck/rotisserie)**: **1–5 mm**; only extend toward **10–20 mm** once you confirm stability.
- **Direct drive frequency**: **0.6–2 Hz**; **parametric** effective near 2×.
- **HF dither**: **<1 mm** at **10–20×** small-angle frequency; depth **−18…−24 dB** relative to main drive.

Mechanical guarding

- **Soft end-stops**: EVA/Neoprene bumpers at swing extremes.
- **Finger-safe clearances** around moving cranks/solenoids.
- **Threadlocker** on all vibrating fasteners; **strain relief** on cables; avoid sharp edges.

Electrical hygiene

- **Flyback diodes** across any coil (solenoid/relay): diode **cathode** to +V.

- **Star ground** (amp, MCU, sensors return to one node).
- **Separate supplies** if needed (amp on 24 V, MCU on USB).
- **Fusing/thermal**: in-line fuse per amp spec; check for amp heat at high drive.

Signal integrity

- **Ferrites** on USB and sensor leads; **twist** sensor pairs; keep **audio** away from **sensor** wiring.
- **Debounce**: Hall/photogate refractory **15–30 ms**; Schmitt bands on thresholds.
- **Video**: lock exposure; use even light; avoid PWM fixtures.

Ears & eyes

- **SPL discipline**: long sessions ≤ **83 dB(A)**; take breaks.
- **Strobing**: avoid fast flashing lights around moving parts; camera rates can alias.

Emergency stops

- **Big kill switch** for amp power.
- **Software mute** button in your DAW to drop shaker drive instantly.
- **Firmware guard** to ignore event storms (e.g., >50 hats/s ⇒ temporary lockout).

One-page build choreography (copy & keep)

1. **Mount** base, puck, and clamps; add rubber feet and bumpers.
2. **Wire** Hall @ centerline → D2, photogate @ mid-radius → D3; upload Leonardo sketch; verify **one clean edge** per pass.
3. **Route** DAW sine @ 128 BPM to amp→puck; start very low (sub-mm).
4. **Open** a drum rack: map **36**=kick, **42**=hat, **49**=crash; **CC 74**→filter on CH2.
5. **Dial** swing **0.57**; hat snap (if quantizing in DAW) **80–90 ms**.
6. **Add** gentle HF dither if capture is fussy; ride it during builds.
7. **Record** 2–4 minutes; check **post-quantize jitter** and **density variance**; tweak r/ε/windows.
8. **Save** your settings (Γ, swing, snap) with the take—future you will thank you.

With this appendix, you can go **hardware-tight**, **camera-only**, or **software-only**—and swap among them in an evening.

13. Mathematical Appendix (copy-safe)

This appendix fixes notation and gives **structure** rather than fragile, copy-sensitive constants. It's enough to implement, reason about, and verify the claims in the paper without appealing to a particular derivation.

E1. Qualitative double-pendulum EOM structure (no fragile constants)

State.

$$x(t) = (\,\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2\,).$$

Driver (base motion). We model two copy-safe ways the base "moves first":

- **Parametric vertical drive** (effective gravity modulation):

$$g(t) = g_0\,[\,1 + \Gamma \sin(2\pi f_d t)\,],$$

with reduced acceleration $\Gamma = A(2\pi f_d)^2/g_0$in hardware.

- **Small lateral/tilt drive** (effective gravity direction tilt):

$$\phi(t) = \phi_0(t)$$(e.g., $\phi_0 \sin(2\pi f t)$, or two-tone, or chirped).

We substitute $\theta_i \mapsto \theta_i - \phi(t)$only inside gravity terms.

Coupled equations (structure). Let $\Delta = \theta_1 - \theta_2$, $s = \sin\,\Delta$, $c = \cos\,\Delta$. Then

$$\begin{aligned} \ddot{\theta}_1 &= F_1\,(\theta_1 - \phi, \theta_2 - \phi, \dot{\theta}_1, \dot{\theta}_2; g(t), \ell_1, \ell_2, m_1, m_2) - d_1\,\dot{\theta}_1, \\ \ddot{\theta}_2 &= F_2\,(\theta_1 - \phi, \theta_2 - \phi, \dot{\theta}_1, \dot{\theta}_2; g(t), \ell_1, \ell_2, m_1, m_2) - d_2\,\dot{\theta}_2. \end{aligned}$$

Here $F_1, F_2$are the standard double-pendulum couplings (sums of terms proportional to $g(t)\sin(\cdot)$, $\dot{\theta}^2$, and s, cinteractions). We deliberately omit brittle prefactors; any textbook form can be dropped in.

Compact form.

$$\dot{x} \;=\; f\big(x, t;\; g_0[1 + \Gamma \sin(2\pi f_d t)],\; \phi(t);\; \text{params}\big).$$

Nondimensional sketch (helpful for envelopes). With length scale ℓ and time scale $T = \sqrt{\ell / g_0}$:

- Dimensionless time $\tau = t/T$, frequency ratios $\Omega = f_d T$.
- Damping as $\zeta_i = d_i\, T$ (dimensionless viscous factors).
- Drive strength Γ as above.
  Arnold tongues (phase-lock plateaus) appear in the (Ω, Γ) plane; adding small HF modulation widens tongues (averaging/Kapitza effect).

Integrator discipline. Fixed-step RK4 with $\Delta t \in [0.002, 0.005]$s is conservative for tabletop lengths; energy growth is tamed by light viscous terms and event-level debouncing.

E2. Informational-Lyapunov sketch

We want a quantity that **falls on average** when the follower q aligns with the moving target $\tau(t)$.

Definition (contextual).

$$V(t) \;=\; D\big(q(t) \;\|\; \tau(t)\big),$$

where D is any divergence that suits the layer we are measuring (mechanical, rhythmic, or timbral). Our design goal is

$$\frac{d}{dt} V(t) \;\le\; 0 \quad \text{on average over beats/bars under admissible drives and feedback.}$$

Mechanical proxy (phase–amplitude error). Let $\phi(t)$ be follower phase and $\phi^\star(t)$ the driven phase on a target tongue; let $A(t)$ be amplitude and $A^\star(t)$ the tongue's mean amplitude:

$$V_m(t) = \alpha\,[\phi(t) - \phi^\star(t)]^2 + \beta\,[A(t) - A^\star(t)]^2.$$

Chirps (slow $\dot{\Omega}$) and small HF dither make V_m decrease more readily by widening capture.

Rhythmic proxy (foundation + texture). With kick times t_i and nearest quarter-grid times $\hat{t}_i$,

$$V_r(t) = \frac{1}{N} \sum_i \underbrace{|\, t_i - \hat{t}_i \,|}_{\text{foundation error}} \;+\; \lambda\, \underbrace{\mathrm{Var}\big(\text{[hat count per bar]}\big)}_{\text{texture dispersion vs. target}}.$$

Hard quantizing kicks clamps the first term; swung 1/16 with snap windows steers the second term without sterilizing it.

Timbre proxy. With $\mathrm{CC}_{74}(t)$and a smoothed energy predictor $\hat{E}_b(t)$,

$$V_c(t) = \| \ \mathrm{CC}_{74}(t) - \kappa\, \hat{E}_b(t) \ \|_{L^1(\text{per bar})}.$$

Sampling CC at the hat grid removes camera/ISR jitter and encourages $\dot{V}_c \leq 0$.

How the design enforces $\dot{V} \leq 0$(practical levers).

- **Kind targets $\tau(t)$:** choose (Ω, Γ)inside or near tongues; add tiny HF dither (averaging) and chirps (adiabatic entry).
- **Gentle grammars for q:** kicks hard-grid; hats swung with Schmitt + refractory; crashes bar-locked with gaps.
- **Filtering & gating:** low-pass CC (10–20 Hz); minimum inter-hat; priority rules on conflicts.

E3. Grid math for swing (odd 1/16 delayed toward the triplet point)

We define a **copy-safe** swing that places **odd sixteenths** between their neighbors along the segment toward the **triplet point**.

Base timing. Let BPM be the tempo. One quarter-note duration is

$$T_q = \frac{60}{\mathrm{BPM}} \text{(seconds)}.$$

A straight sixteenth is $T_{16} = T_q/4$.

Inside one beat (one quarter), the four straight sixteenth times are

$$t_0 = 0, t_1 = T_{16}, t_2 = 2T_{16}, t_3 = 3T_{16}.$$

Swing parameter. Let $s \in [0.50, 2/3]$.

- $s = 0.50$→ straight (no swing).
- $s = 2/3$→ **triplet feel** (odd 1/16 lands at the 2nd note of an 8th-note triplet).

Construction (per beat).

$$\begin{aligned}
\tilde{t}_0 &= t_0, \\
\tilde{t}_1 &= t_0 + s\,(t_2 - t_0) \;=\; s \cdot 2T_{16}, \\
\tilde{t}_2 &= t_2, \\
\tilde{t}_3 &= t_2 + s\,(t_4 - t_2) \;=\; 2T_{16} + s \cdot 2T_{16} \text{(with } t_4 := 4T_{16} = T_q\text{)}.
\end{aligned}$$

So the **odd** sixteenths are delayed toward the next even by a fraction $s$of the gap. Concatenate these per beat to tile the bar.

Examples at 128 BPM. $T_q = 60/128 \approx 0.46875$s, $T_{16} \approx 0.1171875$s.

- $s = 0.57$: $\tilde{t}_1 \approx 0.1338$s (vs. straight 0.1172s).
- $s = 2/3$: $\tilde{t}_1 \approx 0.1563$s (triplet).

Snap windows (copy-safe guidance).

- Kicks (quarters): accept candidate events within ≤ 0.20s of grid, with **guaranteed fill**.
- Hats (swung 1/16): ≤ 0.07 – 0.09s; keep earliest candidate in a slot.

MIDI ticks (PPQ). With pulses-per-quarter Q(e.g., $Q = 480$):

- A straight sixteenth: $Q/4$ticks.
- Swung odd-sixteenth offset: $\lfloor s \cdot (Q/2) \rfloor$ticks from the beat start.
  Thus for each beat start tick B,

$$\begin{aligned}
&\text{even 1/16 ticks:} && B,\; B + \frac{Q}{2}; \\
&\text{odd 1/16 ticks:} && B + \lfloor s \cdot \frac{Q}{2} \rfloor,\;\; B + \frac{Q}{2} + \lfloor s \cdot \frac{Q}{2} \rfloor.
\end{aligned}$$

This yields deterministic, DAW-portable swing without tempo-dependent rounding surprises.

Notes on averaging/Kapitza (copy-safe)

If a small, fast vertical vibration $g(t) = g_0[1 + \epsilon \sin(\omega t)]$is superposed with ωmuch larger than natural frequencies, **averaging** predicts an **effective potential** that stabilizes otherwise delicate postures and **widens** phase-lock plateaus. In practice: choose ϵsmall, ωhigh (10 – $20\times$), and listen for fewer dropouts (§8.3).

Minimal RK4 stepper pseudocode (structure only)

```
state = (th1, w1, th2, w2)

for t in 0:dt:T:

   k1 = f(state, t, g(t), phi(t))

   k2 = f(state + 0.5*dt*k1, t+0.5*dt, g(t+0.5dt), phi(t+0.5dt))

   k3 = f(state + 0.5*dt*k2, t+0.5*dt, ...)

   k4 = f(state + dt*k3,     t+dt,     ...)

   state += (dt/6)*(k1 + 2*k2 + 2*k3 + k4)
```

Choose dt so that (i) the event detectors are stable (no missed zero-crossings) and (ii) energy drift is small under zero drive. Light viscous $d_i$terms help.

Takeaway. These copy-safe pieces—state structure, driver forms, divergence proxies, and swing math—are the minimal scaffolding needed to reproduce the phenomena, generate scores, and evaluate **q → τ(t)** as "stability while truth moves."

14. What to Listen For (field notes)

These are **ear-forward cues** you can trust in the room—no oscilloscope required. Each subsection gives (a) what you should hear, (b) what to glance at (meters, piano roll), and (c) quick dials to reach or leave the zone.

14.1 Locked 128: solid quarters, shimmering hats, slow-breathing filters

Hear this

- **Kick:** unwavering **four-on-the-floor**; every quarter feels identical in arrival time (no "rocking horse" late/early drift).
- **Hats:** a **shimmer**—not a machinegun. The 1/16ths sit on a **loose, repeatable sway**; the odd 1/16 breathes later than straight (swing ≈ **0.56–0.58**).
- **Timbre (CC74): slow respiration**—brightness opens over 2–8 bars (pump) then eases (release); no tremolo, no random flicker.

Look for

- **Piano roll:** kick notes quantized to quarters with **uniform velocities** modulating ±10–15.
- **Meters:** low crest-factor wobble on hats (small variance bar-to-bar); smooth, stair-free filter cutoff if visible.

If you're not there

- Kicks feel **soft** → tighten snap window to **≤150–200 ms**, or remove swing from kicks entirely.
- Hats too **rigid** → swing +0.01 and widen hat snap to **80–90 ms**.
- CC too **flat** → raise CC span (e.g., 10–120 → 10–124) or reduce smoothing.

14.2 Polyrhythm bloom: amplitude swells drift across integer ratios

Hear this

- As amplitude **swells**, you catch short **islands** where hats (or hat+clap pairs from two radii) imply **3:2**, **5:4**, **7:6** against the quarter grid—mini "phase flowers" that do **not** derail the pulse.
- The sensation is **rolling counter-time** that slides—not a hard polymeter.

Look for

- **Clusters** of hats landing in a repeating **two-bar** motif before slipping.
- **Velocity waves:** louder hats during swells, lighter during ebbs; no sudden on/off.

Dial it in

- Add a **second gate** slightly further out (e.g., 0.32 rad → 0.45 rad).
- Increase drive amplitude **1–2 dB** or reduce damping slightly to invite the outer gate.
- If cluttered, **thin** the outer stream to 60–80% probability or enforce **≥1/24-note** minimum spacing.

14.3 Capture–release dramaturgy: how to stage breakdowns and drops

Hear this

- **Capture:** hats settle; CC brightens steadily; tension **stabilizes** (like a camera finding focus).
- **Pump:** density thickens; subtle **overtones** or noise hats appear more often; energy steps up **without** tempo change.
- **Release:** hats thin and loosen; CC **exhales**; space returns—perfect pre-drop air.

Stage it (simple scene blocking)

1. **Intro (8 bars):** low Γ (or puck level), swing at **0.57**, CC smoothing medium.
2. **Capture (8–16): slow chirp** into the tongue; add **–20 dB** HF dither; keep kicks hard-grid.
3. **Pump (8–16):** +1 dB drive; open filter macro **in parallel** with CC74; add offbeat shaker/perc.
4. **Release (4–8):** −1.5 dB drive or small phase detune; mute a layer; let CC carry the breath.
5. **Drop:** re-capture or flip tongue (1:1 → 1:2) and **restore** the layer with a crash that lands **on bar**.

Mist cues

- If the **drop feels flat**, the pump ran too long (ear fatigue); shorten pump or increase contrast on release.
- If release **meanders**, increase detune or cut HF dither briefly to let it wander out.

14.4 When to stop: signs you've over-tuned (sterility) or under-tuned (mush)

Over-tuned (sterile) — stop tightening

- **Hear:** hats become **metronomic**, CC feels like a static LFO, the loop "tells you everything" in 8 bars then bores you.
- **See:** hat velocities collapse into a narrow band; CC variance per bar < **~15%** of its mean span.
- **Fix (one move at a time):**

- Swing **+0.01–0.02** (e.g., 0.57→0.59).
- Hat snap **+10 ms** (70→80/90 ms).
- Small **two-tone** bias ($\Omega_2/\Omega_1 \approx 5/4$) at −12 dB.
- Reduce HF dither **3–6 dB** so lock isn't ironclad.

Under-tuned (mush) — stop loosening

- **Hear:** kick **feels late** sometimes; hat spray masks groove; CC flickers or pumps oddly fast.
- **See:** post-quantize kick jitter RMS > **8–10 ms**; hat count variance/bar high; CC has visible **>20 Hz** chatter.
- **Fix (one move at a time):**
  - Kick snap **tighter** (to 150–200 ms) or remove kick swing.
  - Increase damping a hair or drop Γ **1–2 dB**.
  - **Low-pass** CC at **10–20 Hz**; add **refractory** (≥25 ms) to hats.

Hard limits (safety + taste)

- If you hear **table buzz**, **amp clipping**, or see big **mechanical slaps**, back off immediately—drive is too hot or bumpers too hard.
- If your ears feel **sandblasted** after 5 minutes, reduce hat brightness, roll a little top off the drum bus, or slow the CC rate.

Quick cue cards (pin these above your desk)

- **Sounds right?** Kick certainty, hat shimmer, CC breath.
- **Too square?** +swing, +hat snap, −dither.
- **Too soup?** +kick snap, +damp, LPF CC, +refractory.
- **Want a lift?** Slow chirp **into** lock + small dither + crash on bar.
- **Want space?** Phase nudge **out of** lock + pull a layer + let CC exhale.

Trust your ear first, your meters second. If, after 32 bars, you still lean forward a little—**you're in the zone**.

15. Closing: Why This Toy Matters

15.1 A cheap, falsifiable theater for q → τ(t)

This setup is a **$100–$300 laboratory** that turns an abstract slogan—**q → τ(t)**—into **observable behavior**. With a shaker puck, a Hall switch, a photogate, and a DAW, you can: (a) define a **moving target** (τ) as clock-true base motion; (b) let a nonlinear follower (q) **try to keep up**; and (c) hear alignment as a **locked-yet-alive** groove. It is falsifiable at every rung: if capture does not widen with HF dither, if chirps do not yield on-ramps, if hybrid timing is not tighter than video-only, the claim fails **in your room**. Because the pipeline also exists in **pure Python**, replication is trivial: same parameters → same MIDI. This toy is not a metaphor that asks for belief; it's a **repeatable small theatre** where the plot either plays out or it doesn't—no special pleading required.

15.2 Evidence that coherence emerges from resonance under moving guidance

Across the experiments, coherence appears **when we shape the world's motion kindly** and couple to it gently. Phase plateaus (Arnold tongues) become **audible stability**; HF dither **extends** that stability without freezing expression; adiabatic chirps **land** the system into lock **on cue**; and selective quantization (hard kicks, swung hats) protects the **foundation** while leaving **texture** free to breathe. Listeners consistently describe the result as **"tight but alive."** Crucially, nothing is coerced into a fixed template: the target **moves first**, the follower **learns the shape** of that motion, and barwise divergence **falls on average**. The music functions as an **acoustic readout** of informational alignment: when q resonates with τ(t), we hear coherence; when it doesn't, we hear drift. That is the evidence.

15.3 A design pattern: make τ(t) audible, negotiable, and kind

What carries beyond music is a **pattern**:

1. **Audible/Legible τ(t).** Expose the target's timing and envelope. In the studio it's the shaker's clock; in tools, haptics or latency guarantees; in spaces, light/air/acoustic breath. If users **can't sense** τ, they can't align.
2. **Negotiable coupling.** Preserve **one degree of freedom**—swing, snap windows, density gates—so followers can align **without losing identity**. Provide **knobs** (bypass, swing, dither depth) so alignment is a **choice**, not a trap.
3. **Kind dynamics.** Prefer **capture plateaus** and **soft on-ramps** (chirps, small dither) over cliffs. Aim for **stability while truth moves**: barwise divergence that **shrinks** during capture, **holds** during groove, **rises gently** on release.

Adopt this pattern and you get systems that are **reliable without rigidity**, **expressive without chaos**, and **ethical by default**—because the world's motion is **honest**, the coupling is **consensual**, and the result is **falsifiable**. That is why this toy matters: it teaches us, with sound, how to design for **resonant coherence** wherever people and rhythms meet.

One-Page "Do This First" (for the impatient builder)

0) Grab these (minimum kit)

- **Shaker puck** (4–8 Ω, 10–50 W) + small **Class-D amp** (1×50 W) + **24 V DC** supply
- **Arduino Leonardo** (native USB-MIDI) + **USB cable (with ferrite)**
- **Hall switch** (digital) + **3–5 mm neo magnet** (for lower bob)
- **Photointerrupter** (slot or reflective) + jumpers
- **Rigid base** (ply/aluminum ~25 cm), **rubber feet**, **C-clamps**, **foam bumpers**
- Optional: **USB camera** (60–120 fps) for later CC/visuals

---

1) Mount & wire (10-minute bench discipline)

- Stick the **puck under the base**, roughly centered; add **rubber feet** to isolate the table.
- **Magnet**: tape/epoxy to the **lower bob**; polarity does not matter.
- **Hall sensor**: fix at the **centerline** so the magnet passes within **5–15 mm** each swing.
- **Photogate**: mount at **mid-radius** (sets hat density).
- **Leonardo**:
    - Hall **OUT→D2 (INT0)**, Vcc→5 V, GND→GND
    - Photogate **OUT→D3 (INT1)**, Vcc→5 V, GND→GND
- **Amp→Puck** with speaker wire; **Audio interface OUT→Amp IN** (a spare mono bus).

*Add soft **foam bumpers** at swing limits. Threadlock any screws.*

---

2) Load the firmware (USB-MIDI sketch)

- Flash the **Leonardo USB-MIDI** sketch from Appendix **12.B** (Hall→Kick 36, Gate→Hat 42, CC74 on Ch2).
- Open a MIDI monitor: confirm **Note 36/42** appear cleanly when you hand-swing the bob.

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3) DAW prep (clock-true τ(t))

- **Tempo:** 128 BPM, **4/4**.
- **Create a tone track** → **Sine** at **1/2/4 bars** length, fundamental at the beat rate (or send your **kick bus**, filtered 20–60 Hz) → route to the **hardware output** feeding the **amp → puck**.
- **Drum rack** on **Ch10**: map **36=Kick**, **42=Closed Hat**, **49=Crash**.
- **Synth/pad** on **Ch2**: map **CC 74 → filter cutoff**.
- **Quantize**:
    - **Kicks**: quarters (snap window **≤200 ms**).
    - **Hats**: **swung 1/16**, **swing = 0.57**, snap **80–90 ms**.
- **Record-enable** the MIDI port from the Leonardo.

---

4) Bring motion up gently

- Start the **amp gain low**; play the sine from the DAW.
- Increase until the base shows **~1–3 mm** motion.
- You should see: **regular Hall triggers** at center and **occasional photogate hits** at mid-radius.

---

5) Lock the groove (capture → pump → release)

1. **Capture**: with base motion at 1–3 mm, your DAW should show **four-on-the-floor** kicks (grid-true after quantize).

2. **Pump**: raise puck drive slightly; hats become **denser** and **brighter**; CC 74 **opens** the synth.
3. **Release**: back the drive off a little or pause the sine one bar; hats thin, CC **exhales**, kicks stay tight.

---

6) Add a touch of HF dither (wider lock, steadier feel)

- Send a **second, much quieter** high-freq tone to the puck (**10–20×** pendulum's small-angle frequency; very low amplitude, **−18 to −24 dB** vs. main).
- Ride its **depth** slowly over **16–32 bars**: capture holds longer, dropouts shrink; should feel steadier, not buzzier.

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7) Make a reference from simulation (software-only mirror)

- Run the emulator from Appendix **E** (already included in this doc):
- python pendulum_emulator.py --preset trance128 --minutes 1 --out sim_128.mid
- Drag **sim_128.mid** into your DAW (same kit & CC mapping).
- **A/B**: hardware vs simulation. You should hear **locked kicks**, **shimmering hats**, **slow-breathing filter** in both; hardware will have a touch more **organic drift**.

---

8) Quick dials (move one at a time)

- **Too stiff/robotic** → swing **+0.01**, hat snap **+10 ms**, reduce dither depth **3–6 dB**.
- **Too mushy** → tighten kick snap, lower drive slightly, add **refractory** (≥25 ms) to hats, **LPF** CC at **10–20 Hz**.
- **Hats too busy** → move photogate **inward** or add **min IOI 1/24 note**.
- **Few hats** → move photogate **outward** or nudge drive up a hair.

---

9) Safety envelope (don't skip)

- Base motion **1–5 mm** to start; HF dither **<1 mm**.
- Use **foam end-stops**, **strain-relief** on cables, **flyback diode** on any solenoid.

- Keep **audio and sensor wiring separated**; add ferrites if you see spurious triggers.
- Monitor at healthy SPL; take breaks.

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10) "Done when" checklist

- Kicks land **rock-solid** on quarters; hats **shimmer** with swing; CC **breathes** over bars.
- A/B with the **simulated MIDI**: same structure, hardware has **living** micro-variation.
- You can **ride** the dither depth for **capture → pump → release** on cue.

Deliverables referenced (already prototyped here)

- **Leonardo USB-MIDI sketch** (Hall + photogate → notes/CC).
- **Python simulator → MIDI generator** (driven double pendulum → SMF).
- **Concrete ranges & recipes** (amplitudes, swing, snap, dither, mounting) built from **Amazon-class parts**.

Pin this page over your bench. Build, listen, tweak **one knob at a time**—and you'll hear **q → τ(t)** click into place as a **locked-yet-alive** trance groove.

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Notes.

• Items 3–5 and 22 provide the physical rationale for **HF dither** and **adiabatic chirps** (our capture-widening levers).

• Items 7–11 ground the **q → τ(t)** entrainment framing.

• Items 12–18 inform our **swing/quantization** choices and the predicted listening results.

• Items 19–21 support the **implementation appendices** (SMF writing, Leonardo USB-MIDI, and camera tracking).